Belarusian National Olympiad 2011

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by proximo

– Day 1

– Real nonzero numbers $a, b, c (b > 0)$ satisfy the condition that two distinct roots of the equation $ax^2 + bx - c = 0$ are also roots of the equation $x^3 + bx^2 + ax - c = 0$. Prove the inequalities:
a) $abc > 16$
b)$abc \geq \frac{3125}{108}$

– Find $\{\frac{2009!}{2011}\}$ where $\{x\}$ is a fractional part of number x

– Let M be a midpoint of the side AB of the oxygon $\triangle ABC$, points P and Q are bases of altitudes AP and BQ of this triangle. It is known that circumcircle of $\triangle BMP$ tangents side AC. Prove that circumcircle of $\triangle AMQ$ tangents line BC.

– What is the least number N of 4-digits numbers compiled from digits $1, 2, 3, 4, 5, 6, 7, 8$ you need to choose, that for any two different digits, both of this digits are in
a) At least in one of chosen N numbers?
b)At least in one, but not more than in two of chosen N numbers?

– Day 2

– Let B and C be the points on hyperbola $y = 1/x \ (x > 0)$ and abscissa of point C is greater than abscissa of point B. Line OA (O is an origin) intersects hyperbola $y = 1/x \ (x < 0)$ at point A. Prove that the angle BAC equals one frome the angles between line BC and tangent to hyperbola at point B

– Prove that there exist infinitely many natural numbers n, such that n and the sum of its digits $S(n)$ are perfect squares and there are no digits 0 in n

– Signs $" + "$ or $" - "$ are in each cell of table $n * n$. In one turn it is allowed to reverse all signs in one column or in one row. At the beginning there were two signs $" - "$, and in other cells - $" + "$. After some turns a table with nine signs $" - "$, and in other cells - $" + "$ was obtained. Find the maximum and the minimum values of n.

– Let I be an incenter of non-isosceles oxygon $\triangle ABC$ and Q is a tangent point lying on AB. Point T belongs to side AB and $IT||CQ$. Line TK tangents inscribed circle at the point K (different from the point Q and intersects lines CA and CB at points L and N respectively. Prove that T is a midpoint of LN.