Serbia National Math Olympiad 2016

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–	Day 1
1	Let $n > 1$ be an integer. Prove that there exist $m > n^n$ such that $\frac{n^m - m^n}{m+n}$ is a positive integer.
2	Let n be a positive integer. Let f be a function from nonnegative integers to themselves. Let $f(0,i) = f(i,0) = 0$, $f(1,1) = n$, and $f(i,j) = [\frac{f(i-1,j)}{2}] + [\frac{f(i,j-1)}{2}]$ for positive integers i, j such that $i * j > 1$. Find the number of pairs (i,j) such that $f(i,j)$ is an odd number.($[x]$ is the floor function).
3	Let ABC be a triangle and O its circumcentre. A line tangent to the circumcircle of the triangle BOC intersects sides AB at D and AC at E. Let A' be the image of A under DE. Prove that the circumcircle of the triangle $A'DE$ is tangent to the circumcircle of triangle ABC.
–	Day 2
4	Let ABCbe a triangle, and I the incenter, M midpoint of BC, D the touch point of incircle and BC. Prove that perpendiculars from M, D, A to AI, IM, BC respectively are concurrent
5	There are $2n-1$ twoelement subsets of set $1, 2, ..., n$. Prove that one can choose n out of these such that their union contains no more than $\frac{2}{3}n + 1$ elements.
6	Let $a_1, a_2, \ldots, a_{2^{2016}}$ be positive integers not bigger than 2016. We know that for each $n \leq 2^{2016}$, $a_1 a_2 \ldots a_n + 1$ is a perfect square. Prove that for some i , $a_i = 1$.