Serbia Additional Team Selection Test 2016

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1 Let $P_0(x) = x^3 - 4x$. Sequence of polynomials is defined as following: $P_{n+1} = P_n(1+x)P_n(1-x) - 1$.

Prove that $x^{2016} | P_{2016}(x)$.

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2 Let $ABCD$ be a square with side 4. Find, with proof, the biggest k such that no matter how we place k points into $ABCD$, such that they are on the interior but not on the sides, we always have a square with sidr length 1, which is inside the square $ABCD$, such that it contains no points in its interior(they can be on the sides).

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3 Let $w(x)$ be largest odd divisor of x. Let a, b be natural numbers such that $(a, b) = 1$ and $a + w(b+1)$ and $b + w(a+1)$ are powers of two. Prove that $a+1$ and $b+1$ are powers of two.
