Final Round - Korea 2016

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Day 1 March 19, 2016

1 In a acute triangle $\triangle ABC$, denote D, E as the foot of the perpendicular from B to AC and C to AB.
Denote the reflection of E with respect to AC, BC as S, T.
The circumcircle of $\triangle CST$ hits AC at point $X(\neq C)$.
Denote the circumcenter of $\triangle CST$ as O. Prove that $XO \perp DE$.

2 Two integers n, k satisfies $n \geq 2$ and $k \geq \frac{5}{2}n - 1$.
Prove that whichever k lattice points with x and y coordinate no less than 1 and no more than n we pick, there must be a circle passing through at least four of these points.

3 Prove that for all rationals x, y, $x - \frac{1}{x} + y - \frac{1}{y} = 4$ is not true.

Day 2 March 20, 2016

4 If x, y, z satisfies $x^2 + y^2 + z^2 = 1$, find the maximum possible value of

$$(x^2 - yz)(y^2 - zx)(z^2 - xy)$$

5 An acute triangle $\triangle ABC$ has incenter I, and the incircle hits BC, CA, AB at D, E, F.
Lines BI, CI, BC, DI hits EF at K, L, M, Q and the line connecting the midpoint of segment CL and M hits the line segment CK at P. Prove that

$$PQ = \frac{AB \cdot KQ}{BI}$$

6 Let U be a set of m triangles. Prove that there exists a subset W of U which satisfies the following.

(i). The number of triangles in W is at least $0.45m^{\frac{4}{5}}$

(ii) There are no points A, B, C, D, E, F such that triangles ABC, BCD, CDE, DEF, EFA, FAB are all in W.