Macedonian National Olympiad 2016

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Problem 1 Solve the equation in the set of natural numbers $1 + x^z + y^z = LCM(x^z, y^z)$

Problem 2 A magic square is a square with side 3 consisting of 9 unit squares, such that the numbers written in the unit squares (one number in each square) satisfy the following property: the sum of the numbers in each row is equal to the sum of the numbers in each column and is equal to the sum of all the numbers written in any of the two diagonals.

A rectangle with sides $m \geq 3$ and $n \geq 3$ consists of mn unit squares. If in each of those unit squares exactly one number is written, such that any square with side 3 is a magic square, then find the number of most different numbers that can be written in that rectangle.

Problem 3 Solve the equation in the set of natural numbers $xyz + yzt + xzt + xyt = xyzt + 3$

Problem 4 A segment AB is given and it's midpoint K. On the perpendicular line to AB, passing through K a point C, different from K is chosen. Let N be the intersection of AC and the line passing through B and the midpoint of CK. Let U be the intersection point of AB and the line passing through C and L, the midpoint of BN. Prove that the ratio of the areas of the triangles CNL and BUL, is independent of the choice of the point C.

Problem 5 Let $n \geq 3$ and $a_1, a_2, ..., a_n \in \mathbb{R}^+$, such that $\frac{1}{1+a_1^4} + \frac{1}{1+a_2^4} + ... + \frac{1}{1+a_n^4} = 1$. Prove that:

$$a_1 a_2 ... a_n \geq (n-1)^{\frac{n}{4}}$$
