Lexington Math Tournament

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by parmenides51, kevinmathz

–	Divisions
A1	Triangle LMT has $\overline{MA}$ as an altitude. Given that $MA = 16$, $MT = 20$, and $LT = 25$, find the length of the altitude from L to $\overline{MT}$. *Proposed by Kevin Zhao*
A2	The function $f(x)$ has the property that $f(x) = -\frac{1}{f(x-1)}$. Given that $f(0) = -\frac{1}{21}$, find the value of $f(2021)$. *Proposed by Ada Tsui*
A3	Find the greatest possible sum of integers a and b such that $\frac{2021!}{20^a \cdot 21^b}$ is a positive integer. *Proposed by Aidan Duncan*
A4 B11	Five members of the Lexington Math Team are sitting around a table. Each flips a fair coin. Given that the probability that three consecutive members flip heads is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$. *Proposed by Alex Li*
A5	In rectangle $ABCD$, points E and F are on sides $\overline{BC}$ and $\overline{AD}$, respectively. Two congruent semicircles are drawn with centers E and F such that they both lie entirely on or inside the rectangle, the semicircle with center E passes through C, and the semicircle with center F passes through A. Given that $AB = 8$, $CE = 5$, and the semicircles are tangent, find the length BC. *Proposed by Ada Tsui*
A6 B12	Given that the expected amount of 1s in a randomly selected 2021-digit number is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$. *Proposed by Hannah Shen*
A7 B15	A geometric sequence consists of 11 terms. The arithmetic mean of the first 6 terms is 63, and the arithmetic mean of the last 6 terms is 2016. Find the 7th term in the sequence. *Proposed by Powell Zhang*

A8 Isosceles $\triangle ABC$ has interior point O such that $AO = \sqrt{52}$, $BO = 3$, and $CO = 5$. Given that $\angle ABC = 120^\circ$, find the length AB.

Proposed by Powell Zhang

A9 Find the sum of all positive integers n such that $7 < n < 100$ and 1573_n has 6 factors when written in base 10.

Proposed by Aidan Duncan

A10 Pieck the Frog hops on Pascal's Triangle, where she starts at the number 1 at the top. In a hop, Pieck can hop to one of the two numbers directly below the number she is currently on with equal probability. Given that the expected value of the number she is on after 7 hops is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

Proposed by Steven Yu

A11 B17 In $\triangle ABC$ with $\angle BAC = 60^\circ$ and circumcircle ω, the angle bisector of $\angle BAC$ intersects side $\overline{BC}$ at point D, and line AD is extended past D to a point A'. Let points E and F be the feet of the perpendiculars of A' onto lines AB and AC, respectively. Suppose that ω is tangent to line EF at a point P between E and F such that $\frac{EP}{FP} = \frac{1}{2}$. Given that $EF = 6$, the area of $\triangle ABC$ can be written as $\frac{m\sqrt{n}}{p}$, where m and p are relatively prime positive integers, and n is a positive integer not divisible by the square of any prime. Find $m+n+p$.

Proposed by Taiki Aiba

A12 B18 There are 23 balls on a table, all of which are either red or blue, such that the probability that there are n red balls and $23-n$ blue balls on the table ($1 \le n \le 22$) is proportional to n. (e.g. the probability that there are 2 red balls and 21 blue balls is twice the probability that there are 1 red ball and 22 blue balls.) Given that the probability that the red balls and blue balls can be arranged in a line such that there is a blue ball on each end, no two red balls are next to each other, and an equal number of blue balls can be placed between each pair of adjacent red balls is $\frac{a}{b}$, where a and b are relatively prime positive integers, find $a+b$. Note: There can be any nonzero number of consecutive blue balls at the ends of the line.

Proposed by Ada Tsui

A13 In a round-robin tournament, where any two players play each other exactly once, the fact holds that among every three students A, B, and C, one of the students beats the other two. Given that there are six players in the tournament and Aidan beats Zach but loses to Andrew, find how many ways there are for the tournament to play out. Note: The order in which the matches take place does not matter.

Proposed by Kevin Zhao

A14 Alex, Bob, and Chris are driving cars down a road at distinct constant rates. All people are driving a positive integer number of miles per hour. All of their cars are 15 feet long. It takes Alex 1 second longer to completely pass Chris than it takes Bob to completely pass Chris. The passing time is defined as the time where their cars overlap. Find the smallest possible sum of their speeds, in miles per hour.

Proposed by Sammy Charney

A15 B20 Andy and Eddie play a game in which they continuously flip a fair coin. They stop flipping when either they flip tails, heads, and tails consecutively in that order, or they flip three tails in a row. Then, if there has been an odd number of flips, Andy wins, and otherwise Eddie wins. Given that the probability that Andy wins is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

Proposed by Anderw Zhao and Zachary Perry

A16 Find the number of ordered pairs (a, b) of positive integers less than or equal to 20 such that

$$\gcd(a,b) > 1 \quad \text{and} \quad \frac{1}{\gcd(a,b)} + \frac{a+b}{\text{lcm}(a,b)} \geq 1.$$

Proposed by Zachary Perry

A17 Given that the value of

$$\sum_{k=1}^{2021} \frac{1}{1^2+2^2+3^2+\cdots+k^2} + \sum_{k=1}^{1010} \frac{6}{2k^2-k} + \sum_{k=1011}^{2021} \frac{24}{2k+1}$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

Proposed by Aidan Duncan

A18 Points X and Y are on a parabola of the form $y = \frac{x^2}{a^2}$ and A is the point $(x, y) = (0, a)$. Assume XY passes through A and hits the line $y = -a$ at a point B. Let ω be the circle passing through $(0, -a)$, A, and B. A point P is chosen on ω such that $PA = 8$. Given that X is between A and B, $AX = 2$, and $XB = 10$, find $PX \cdot PY$.

Proposed by Kevin Zhao

A19 Let S be the sum of all possible values of $a \cdot c$ such that

$$a^3 + 3ab^2 - 72ab + 432a = 4c^3$$

if a, b, and c are positive integers, $a + b > 11$, $a > b - 13$, and $c \leq 1000$. Find the sum of all distinct prime factors of S.

Proposed by Kevin Zhao

A20 Let Ω be a circle with center O. Let ω_1 and ω_2 be circles with centers O_1 and O_2, respectively, internally tangent to Ω at points A and B, respectively, such that O_1 is on $\overline{OA}$, and O_2 is on $\overline{OB}$ and ω_1. There exists a point P on line AB such that P is on both ω_1 and ω_2. Let the external tangent of ω_1 and ω_2 on the same side of line AB as O hit ω_1 at X and ω_2 at Y, and let lines AX and BY intersect at N. Given that $O_1X = 81$ and $O_2Y = 18$, the value of $NX \cdot NA$ can be written as $a\sqrt{b}+c$, where a, b, and c are positive integers, and b is not divisible by the square of a prime. Find $a+b+c$.

Proposed by Kevin Zhao

A21 B22 A Haiku is a Japanese poem of seventeen syllables, in three lines of five, seven, and five.

In how many ways
Can you add three integers
Summing seventeen?
Order matters here.
For example, eight, three, six
Is not eight, six, three.
All nonnegative,
Do not need to be distinct.
What is your answer?

Proposed by Derek Gao

A22 B23 A Haiku is a Japanese poem of seventeen syllables, in three lines of five, seven, and five.

Ada has been told
To write down five haikus plus
Two more every hour.
Such that she needs to
Write down five in the first hour
Seven, nine, so on.
Ada has so far
Forty haikus and writes down
Seven every hour.
At which hour after
She begins will she not have
Enough haikus done?

Proposed by Ada Tsui

A23 B24 A Haiku is a Japanese poem of seventeen syllables, in three lines of five, seven, and five.

A group of haikus
Some have one syllable less

Sixteen in total.
The group of haikus
Some have one syllable more
Eighteen in total.
What is the largest
Total count of syllables
That the group can't have?
(For instance, a group
Sixteen, seventeen, eighteen
Fifty-one total.)
(Also, you can have
No sixteen, no eighteen
Syllable haikus)

Proposed by Jeff Lin

A 24 A Haiku is a Japanese poem of seventeen syllables, in three lines of five, seven, and five.

Using the four words
"Hi", "hey", "hello", and "haiku",
How many haikus
Can somebody make?
(Repetition is allowed,
Order does matter.)

Proposed by Jeff Lin

A25 B26 Chandler the Octopus is making a concoction to create the perfect ink. He adds 1.2 grams of melanin, 4.2 grams of enzymes, and 6.6 grams of polysaccharides. But Chandler accidentally added n grams of an extra ingredient to the concoction, Chemical X, to create glue. Given that Chemical X contains none of the three aforementioned ingredients, and the percentages of melanin, enzymes, and polysaccharides in the final concoction are all integers, find the sum of all possible positive integer values of n.

Proposed by Taiki Aiba

A26 B27 Chandler the Octopus along with his friends Maisy the Bear and Jeff the Frog are solving LMT problems. It takes Maisy 3 minutes to solve a problem, Chandler 4 minutes to solve a problem and Jeff 5 minutes to solve a problem. They start at $12:00$ pm, and Chandler has a dentist appointment from $12:10$ pm to $12:30$, after which he comes back and continues solving LMT problems. The time it will take for them to finish solving 50 LMT problems, in hours, is m/n ,where m and n are relatively prime positive integers. Find $m+n$.

Note: they may collaborate on problems.

Proposed by Aditya Rao

A27 Chandler the Octopus is at a tentacle party!
At this party, there is 1 creature with 2 tentacles, 2 creatures with 3 tentacles, 3 creatures with 4 tentacles, all the way up to 14 creatures with 15 tentacles. Each tentacle is distinguishable from all other tentacles. For some $2 \le m < n \le 15$, a creature with m tentacles "meets" a creature with n tentacles; "meeting" another creature consists of shaking exactly 1 tentacle with each other. Find the number of ways there are to pick distinct $m < n$ between 2 and 15, inclusive, and then to pick a creature with m tentacles to "meet" a selected creature with n tentacles.

Proposed by Armaan Tipirneni, Richard Chen, and Denise the Octopus

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A28 B29 Addison and Emerson are playing a card game with three rounds. Addison has the cards $1, 3$, and 5, and Emerson
has the cards $2, 4$, and 6. In advance of the game, both designate each one of their cards to be played for either round one, two, or three. Cards cannot be played for multiple rounds. In each round, both show each other their designated card for that round, and the person with the higher-numbered card wins the round. The person who wins the most rounds wins the game. Let m/n be the probability that Emerson wins, where m and n are relatively prime positive integers. Find $m + n$.

Proposed by Ada Tsui

A29 B30 In a group of 6 people playing the card game Tractor, all 54 cards from 3 decks are dealt evenly to all the players
at random. Each deck is dealt individually. Let the probability that no one has at least two of the same card be X.
Find the largest integer n such that the nth root of X is rational.

Proposed by Sammy Charney

Due to the problem having infinitely many solutions, all teams who inputted answers received points.

A30 Ryan Murphy is playing poker. He is dealt a hand of 5 cards. Given that the probability that he has a straight hand (the ranks are all consecutive; e.g. $3, 4, 5, 6, 7$ or $9, 10, J, Q, K$) or 3 of a kind (at least 3 cards of the same rank; e.g. $5, 5, 5, 7, 7$ or $5, 5, 5, 7, K$) is m/n , where m and n are relatively prime positive integers, find $m + n$.

Proposed by Aditya Rao

B1 Given that the expression $\frac{20^{21}}{20^{20}} + \frac{20^{20}}{20^{21}}$ can be written in the form m/n , where m and n are relatively prime positive integers, find $m + n$.

Proposed by Ada Tsui

B2 Find the greatest possible distance between any two points inside or along the perimeter of an equilateral triangle with side length 2.

Proposed by Alex Li

B3 Aidan rolls a pair of fair, six sided dice. Let$n$ be the probability that the product of the two numbers at the top is prime. Given that n can be written as a/b , where a and b are relatively prime positive integers, find $a+b$.

Proposed by Aidan Duncan

B4 Set S contains exactly 36 elements in the form of $2^m \cdot 5^n$ for integers $0 \le m, n \le 5$. Two distinct elements of S are randomly chosen. Given that the probability that their product is divisible by 10^7 is a/b, where a and b are relatively prime positive integers, find $a+b$.

Proposed by Ada Tsui

B5 Find the number of ways there are to permute the elements of the set $\{1,2,3,4,5,6,7,8,9\}$ such that no two adjacent numbers are both even or both odd.

Proposed by Ephram Chun

B6 Maisy is at the origin of the coordinate plane. On her first step, she moves 1 unit up. On her second step, she moves 1 unit to the right. On her third step, she moves 2 units up. On her fourth step, she moves 2 units to the right. She repeats this pattern with each odd-numbered step being 1 unit more than the previous step. Given that the point that Maisy lands on after her 21st step can be written in the form (x, y), find the value of $x + y$.

Proposed by Audrey Chun

B7 Given that x and y are positive real numbers such that $\frac{5}{x} = \frac{y}{13} = \frac{x}{y}$, find the value of $x^3 + y^3$.

Proposed by Ephram Chun

B8 Find the number of arithmetic sequences a_1, a_2, a_3 of three nonzero integers such that the sum of the terms in the sequence is equal to the product of the terms in the sequence.

Proposed by Sammy Charney

B9 Convex pentagon $PQRST$ has $PQ = TP = 5$, $QR = RS = ST = 6$, and $\angle QRS = \angle RST = 90^o$. Given that points U and V exist such that $RU = UV = VS = 2$, find the area of pentagon $PQUVT$.

Proposed by Kira Tang

B10 Let $f(x)$ be a function mapping real numbers to real numbers. Given that $f(f(x)) = \frac{1}{3x}$, and $f(2) = \frac{1}{9}$, find $f\left(\frac{1}{6}\right)$.

Proposed by Zachary Perry

B13 Call a 4-digit number $\overline{abcd}$ *unnoticeable* if $a+c=b+d$ and $\overline{abcd}+\overline{cdab}$ is a multiple of 7. Find the number of unnoticeable numbers.

Note: a, b, c, and d are nonzero distinct digits.

Proposed by Aditya Rao

B14 In the expansion of $(2x+3y)^{20}$, find the number of coefficients divisible by 144.

Proposed by Hannah Shen

B16 Bob plants two saplings. Each day, each sapling has a $1/3$ chance of instantly turning into a tree. Given that the expected number of days it takes both trees to grow is m/n , where m and n are relatively prime positive integers, find $m+n$.

Proposed by Powell Zhang

B19 Kevin is at the point $(19,12)$. He wants to walk to a point on the ellipse $9x^2+25y^2=8100$, and then walk to $(-24,0)$. Find the shortest length that he has to walk.

Proposed by Kevin Zhao

B21 A Haiku is a Japanese poem of seventeen syllables, in three lines of five, seven, and five.

Take five good haikus
Scramble their lines randomly
What are the chances
That you end up with
Five completely good haikus
(With five, seven, five)?
Your answer will be
m over n where m,n
Are numbers such that
m,n positive
Integers where gcd
Of m,n is 1.
Take this answer and
Add the numerator and
Denominator.

Proposed by Jeff Lin

B28 Maisy and Jeff are playing a game with a deck of cards with 4 0's, 4 1's, 4 2's, all the way up to 4 9's. You cannot tell apart cards of the same number. After shuffling the deck, Maisy and Jeff each take 4 cards, make the largest 4-digit integer they can, and then compare. The person with the larger 4-digit integer wins. Jeff goes first and draws the cards $2, 0, 2, 1$ from the deck. Find the number of hands Maisy can draw to beat that, if the order in which she draws the cards matters.

Proposed by Richard Chen

– Guts Round

– Round 1

p1. How many ways are there to arrange the letters in the word $NEVERLAND$ such that the 2 N's are adjacent and the two E's are adjacent? Assume that letters that appear the same are not distinct.

p2. In rectangle $ABCD$, E and F are on AB and CD, respectively such that $DE = EF = FB$ and $\angle CDE = 45^o$. Find $AB + AD$ given that AB and AD are relatively prime positive integers.

p3. Maisy Airlines sees n takeoffs per day. Find the minimum value of n such that theremust exist two planes that take off within aminute of each other.

Round 2

p4. Nick is mixing two solutions. He has 100 mL of a solution that is 30% X and 400 mL of a solution that is 10% X. If he combines the two, what percent X is the final solution?

p5. Find the number of ordered pairs (a, b), where a and b are positive integers, such that

$$\frac{1}{a} + \frac{2}{b} = \frac{1}{12}.$$

p6. 25 balls are arranged in a 5 by 5 square. Four of the balls are randomly removed from the square. Given that the probability that the square can be rotated 180^o and still maintain the same configuration can be expressed as $\frac{m}{n}$, where m and n are relatively prime, find $m + n$.

Round 3

p7. Maisy the ant is on corner A of a $13 \times 13 \times 13$ box. She needs to get to the opposite corner called B. Maisy can only walk along the surface of the cube and takes the path that covers the least distance. Let C and D be the possible points where she turns on her path. Find $AC^2 + AD^2 + BC^2 + BD^2 - AB^2 - CD^2$.

p8. Maisyton has recently built 5 intersections. Some intersections will get a park and some of those that get a park will also get a chess school. Find how many different ways this can happen.

p9. Let $f(x) = 2x - 1$. Find the value of x that minimizes $|f(f(f(f(f(x))))) - 2020|$.

Round 4

p10. Triangle ABC is isosceles, with $AB = BC > AC$. Let the angle bisector of $\angle A$ intersect side $\overline{BC}$ at point D, and let the altitude from A intersect side $\overline{BC}$ at point E. If $\angle A = \angle C = x^o$, then the measure of $\angle DAE$ can be expressed as $(ax - b)^o$, for some constants a and b. Find ab.

p11. Maisy randomly chooses 4 integers w, x, y, and z, where $w, x, y, z \in \{1, 2, 3, ..., 2019, 2020\}$. Given that the probability that $w^2 + x^2 + y^2 + z^2$ is not divisible by 4 is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m + n$.

p12. Evaluate

$$-\log_4\left(\log_2\left(\sqrt{\sqrt{\sqrt{...\sqrt{16}}}}\right)\right),$$

where there are 100 square root signs.

PS. You should use hide for answers. Rounds 5-8 have been posted here (https://artofproblemsolving.com/community/c3h3166476p28814111) and 9-12 here (https://artofproblemsolving.com/community/c3h3166480p28814155). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

– Round 5

p13. Pieck the Frog hops on Pascal's Triangle, where she starts at the number 1 at the top. In a hop, Pieck can hop to one of the two numbers directly below the number she is currently on with equal probability. Given that the expected value of the number she is on after 7 hops is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

p14. Maisy chooses a random set (x, y) that satisfies

$$x^2 + y^2 - 26x - 10y \leq 482.$$

The probability that $y > 0$ can be expressed as $\frac{A\pi - B\sqrt{C}}{D\pi}$. Find $A+B+C+D$.

Due to the problem having a typo, all teams who inputted answers received points

p15. 6 points are located on a circle. How many ways are there to draw any number of line segments between the points such that none of the line segments overlap and none of the points are on more than one line segment? (It is possible to draw no line segments).

Round 6

p16. Find the number of 3 by 3 grids such that each square in the grid is colored white or black and no two black squares share an edge.

p17. Let ABC be a triangle with side lengths $AB = 20$, $BC = 25$, and $AC = 15$. Let D be the point on BC such that $CD = 4$. Let E be the foot of the altitude from A to BC. Let F be the intersection of AE with the circle of radius 7 centered at A such that F is outside of triangle ABC. DF can be expressed as $\sqrt{m}$, where m is a positive integer. Find m.

p18. Bill and Frank were arrested under suspicion for committing a crime and face the classic Prisoner's Dilemma. They are both given the choice whether to rat out the other and walk away, leaving their partner to face a 9 year prison sentence. Given that neither of them talk, they both face a 3 year sentence. If both of them talk, they both will serve a 6 year sentence. Both Bill and Frank talk or do not talk with the same probabilities. Given the probability that at least one of them talks is $\frac{11}{36}$, find the expected duration of Bill's sentence in months.

Round 7

p19. Rectangle $ABCD$ has point E on side $\overline{CD}$. Point F is the intersection of $\overline{AC}$ and $\overline{BE}$. Given that the area of $\triangle AFB$ is 175 and the area of $\triangle CFE$ is 28, find the area of $ADEF$.

p20. Real numbers x, y, and z satisfy the system of equations

$$5x + 13y - z = 100,$$

$$25x^2 + 169y^2 - z2 + 130xy = 16000,$$

$$80x + 208y - 2z = 2020.$$

Find the value of xyz.

Due to the problem having infinitely many solutions, all teams who inputted answers received points.

p21. Bob is standing at the number 1 on the number line. If Bob is standing at the number n, he can move to $n + 1$, $n + 2$, or $n + 4$. In howmany different ways can he move to the number 10?

Round 8

p22. A sequence $a_1, a_2, a_3, ...$ of positive integers is defined such that $a_1 = 4$, and for each integer $k \geq 2$,

$$2(a_{k-1} + a_k + a_{k+1}) = a_k a_{k-1} + 8.$$

Given that $a_6 = 488$, find $a_2 + a_3 + a_4 + a_5$.

p23. $\overline{PQ}$ is a diameter of circle ω with radius 1 and center O. Let A be a point such that AP is tangent to ω. Let γ be a circle with diameter AP. Let A' be where AQ hits the circle with diameter AP and A'' be where AO hits the circle with diameter OP. Let $A'A''$ hit PQ at R. Given that the value of the length RA' is is always less than k and k is minimized, find the greatest integer less than or equal to $1000k$.

p24. You have cards numbered $1, 2, 3, ..., 100$ all in a line, in that order. You may swap any two adjacent cards at any time. Given that you make $\binom{100}{2}$ total swaps, where you swap each distinct pair of cards exactly once, and do not do any swaps simultaneously, find the total number of distinct possible final orderings of the cards.

PS. You should use hide for answers. Rounds 1-4 have been posted here (`https://artofproblemsolving.com/community/c3h3166472p28814057`) and 9-12 here (`https://artofproblemsolving.com/community/c3h3166480p28814155`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

– Round 9

p25. Let a, b, and c be positive numbers with $a + b + c = 4$. If $a, b, c \leq 2$ and

$$M = \frac{a^3 + 5a}{4a^2 + 2} + \frac{b^3 + 5b}{4b^2 + 2} + \frac{c^3 + 5c}{4c^2 + 2},$$

then find the maximum possible value of $\lfloor 100M \rfloor$.

p26. In $\triangle ABC$, $AB = 15$, $AC = 16$, and $BC = 17$. Points E and F are chosen on sides AC and AB, respectively, such that $CE = 1$ and $BF = 3$. A point D is chosen on side BC, and let the circumcircles of $\triangle BFD$ and $\triangle CED$ intersect at point $P \neq D$. Given that $\angle PEF = 30^o$, the length of segment PF can be expressed as $\frac{m}{n}$. Find $m + n$.

p27. Arnold and Barnold are playing a game with a pile of sticks with Arnold starting first. Each turn, a player can either remove 7 sticks or 13 sticks. If there are fewer than 7 sticks at the start of a player's turn, then they lose. Both players play optimally. Find the largest number of sticks under 200 where Barnold has a winning strategy

<u>Round 10</u>

p28. Let a, b, and c be positive real numbers such that $\log_2(a) - 2 = \log_3(b) = \log_5(c)$ and $a + b = c$. What is $a + b + c$?

p29. Two points, $P(x, y)$ and $Q(-x, y)$ are selected on parabola $y = x^2$ such that $x > 0$ and the triangle formed by points P, Q, and the origin has equal area and perimeter. Find y.

p30. 5 families are attending a wedding. 2 families consist of 4 people, 2 families consist of 3 people, and 1 family consists of 2 people. A very long row of 25 chairs is set up for the families to sit in. Given that all members of the same family sit next to each other, let the number of ways all the people can sit in the chairs such that no two members of different families sit next to each other be n. Find the number of factors of n.

<u>Round 11</u>

p31. Let polynomial $P(x) = x^3 + ax^2 + bx + c$ have (not neccessarily real) roots r_1, r_2, and r_3. If $2ab = a^3 - 20 = 6c - 21$, then the value of $|r_1^3 + r_2^3 + r_3^3|$ can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find the value of $m + n$.

p32. In acute $\triangle ABC$, let H, I , O, and G be the orthocenter, incenter, circumcenter, and centroid of $\triangle ABC$, respectively. Suppose that there exists a circle ω passing through B, I , H, and C, the circumradius of $\triangle ABC$ is 312, and $OG = 80$. Let H', distinct from H, be the point on ω such that $\overline{HH'}$ is a diameter of ω. Given that lines $H'O$ and BC meet at a point P, find the length OP.

p33. Find the number of ordered quadruples (x, y, z, w) such that $0 \le x, y, z, w \le 1000$ are integers and
$$x! + y! = 2^z \cdot w!$$

holds (Note: $0! = 1$).

Round 12

p34. Let Z be the product of all the answers from the teams for this question. Estimate the number of digits of Z. If your estimate is E and the answer is A, your score for this problem will be

$$\max\left(0, \lceil 15 - |A - E|\rceil\right).$$

Your answer must be a positive integer.

p35. Let N be number of ordered pairs of positive integers (x, y) such that $3x^2 - y^2 = 2$ and $x < 2^{75}$. Estimate N. If your estimate is E and the answer is A, your score for this problem will be

$$\max\left(0, \lceil 15 - 2|A - E|\rceil\right).$$

p36. 30 points are located on a circle. How many ways are there to draw any number of line segments between the points such that none of the line segments overlap and none of the points are on more than one line segment? (It is possible to draw no line segments). If your estimate is E and the answer is A, your score for this problem will be

$$\max\left(0, \left\lceil 15 - \ln \frac{A}{E}\right\rceil\right).$$

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