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1 Determine the smallest positive integer q with the following property: for every integer m with $1 \leqslant m \leqslant 1006$, there exists an integer n such that

$$\frac{m}{1007}q < n < \frac{m+1}{1008}q$$

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2 Let ABC be an acute triangle with circumcentre O. Let Γ_B be the circle through A and B that is tangent to AC, and let Γ_C be the circle through A and C that is tangent to AB. An arbitrary line through A intersects Γ_B again in X and Γ_C again in Y. Prove that $|OX| = |OY|$.

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3 Does there exist a prime number whose decimal representation is of the form $3811\cdots11$ (that is, consisting of the digits 3 and 8 in that order, followed by one or more digits 1)?

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4 Let n be a positive integer. For each partition of the set $\{1, 2, \dots, 3n\}$ into arithmetic progressions, we consider the sum S of the respective common differences of these arithmetic progressions. What is the maximal value that S can attain?

(An *arithmetic progression* is a set of the form $\{a, a+d, \dots, a+kd\}$, where a, d, k are positive integers, and $k \geqslant 2$; thus an arithmetic progression has at least three elements, and successive elements have difference d, called the *common difference* of the arithmetic progression.)
