KJMO 2020

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by parmenides51, matheeeee, roughlife, jmsung1004

–	day 1
1	The integer n is a number expressed as the sum of an even number of different positive integers less than or equal to 2000. 1+2+ $\cdots$ +2000 Find all of the following positive integers that cannot be the value of n.
2	Let ABC be an acute triangle with circumcircle Ω and $\overline{AB} < \overline{AC}$. The angle bisector of A meets Ω again at D, and the line through D, perpendicular to BC meets Ω again at E. The circle centered at A, passing through E meets the line DE again at F. Let K be the circumcircle of triangle ADF. Prove that AK is perpendicular to BC.
3	The permutation σ consisting of four words A, B, C, D has $f_{AB}(\sigma)$, the sum of the number of B placed rightside of every A. We can define $f_{BC}(\sigma)$,$f_{CD}(\sigma)$,$f_{DA}(\sigma)$ as the same way too. For example, $\sigma = ACBDBACDCBAD$, $f_{AB}(\sigma) = 3+1+0 = 4$, $f_{BC}(\sigma) = 4$,$f_{CD}(\sigma) = 6$, $f_{DA}(\sigma) = 3$ Find the maximal value of $f_{AB}(\sigma) + f_{BC}(\sigma) + f_{CD}(\sigma) + f_{DA}(\sigma)$, when σ consists of 2020 letters for each A, B, C, D
–	day 2
4	In an acute triangle ABC with $\overline{AB} > \overline{AC}$, let D, E, F be the feet of the altitudes from A, B, C, respectively. Let P be an intersection of lines EF and BC, and let Q be a point on the segment BD such that $\angle QFD = \angle EPC$. Let O, H denote the circumcenter and the orthocenter of triangle ABC, respectively. Suppose that OH is perpendicular to AQ. Prove that P, O, H are collinear.
5	Let a, b, c, d, e be real numbers satisfying the following conditions. $$a \le b \le c \le d \le e, \quad a+e=1, \quad b+c+d=3, \quad a^2+b^2+c^2+d^2+e^2=14$$ Determine the maximum possible value of ae.
6	for a positive integer n, there are positive integers $a_1, a_2, ...a_n$ that satisfy these two. (1) $a_1 = 1, a_n = 2020$ (2) for all integer i, $i$satisfies $2 \le i \le n, a_i - a_{i-1} = -2$ or 3. find the greatest n