2nd competition, an Argentinian geometry contest
www.artofproblemsolving.com/community/c2662364
by parmenides51

– round 1

– level 1

p1. Construct the given figure, where $ABCD$ is a square and AEF is an equilateral triangle.
`https://cdn.artofproblemsolving.com/attachments/f/c/1b4043aeed5992ddb8739eec5a8e72ebf4cf9`
`gif`

p2. Let ABC be an isosceles triangle ($AB = AC$). We draw the perpendicular bisector m of AC and the bisector n of angle $\angle C$. If m, n and AB intersect at a single point, how much is angle $\angle A$?

p3. Let A, B, and C be points on a circle. Let us call the orthocenter of the triangle H. Find the locus of H as A moves around the circle.

level 2

p4. Given 3 points A, B and C, construct the isosceles trapezoid $ABCD$ where $AB = CD$ and BC is parallel to AD (BC different from AD).

p5. Let A, B and C be points on a circle. Let's call the centroid of the triangle G. Find the locus of G as A moves along the circle.

p6. Given a triangle ABC, let D, E, and F be the midpoints of the sides BC, CA, and AB, respectively. From D the lines M_1 and M_2 are drawn, perpendicular on AB and AC respectively. From E the lines M_3 and M_4 are drawn, perpendicular on BC and AB respectively. From F the lines M_5 and M_6 are drawn perpendicular on AC and BC respectively. Let A' be the intersection between M_4 and M_5. Let B' be the intersection between M_6 and M_1. Let C' be the intersection between M_2 and M_3. Show that the triangles ABC and $A'B'C'$ are similar and find the ratio of similarity.

– final round

– level 1

p1. Three points are given O, G and M. Construct a triangle in such a way that O is its circumcenter, G is its centroid, and M is the midpoint of one side.

p2. Let ABC be a triangle and H its orthocenter. The height is drawn from A, which intersects BC at D. On the extension of the altitude AD the point E is taken in such a way that the angles $\angle CAD$ and $\angle CBE$ are equal. Prove that $BE = BH$.

p3. Let ω be a circle, and M be a variable point on its exterior. From M the tangents to ω . Let A and B be the touchpoints. Find the locus of the incenter of the triangle MAB as M varies.

p4. i) Find a point D in the interior of a triangle ABC such that the areas of the triangles ABD, BCD and CAD are equal.

ii) The same as i) but with D outside ABC.

– level 2

p5. Let ABC be a triangle and M be a variable point on AB. N is the point on the prolongation of AC such that $CN = BM$ and that it does not belong to the ray CA. The parallelogram $BMNP$ is constructed (in that order). Find the locus of P as M varies.

p6. Let $ABCD$ be a quadrilateral. Let C_1, C_2, C_3, C_4 be the circles of diameters AB, BC, CD and DA respectively. Let P, Q, R and S be the points of intersection (which are not vertices of $ABCD$) of C_1 and C_2, C_2 and C_3, C_3 and C_4, C_4 and C_1 respectively. Show that the quadrilaterals $ABCD$ and $PQRS$ are similar.

p7. M, N, and P are three collinear points, with N between M and P. Let r be the perpendicualr bisector of NP. A point O is taken over r. ω is the circle with center O passing through N. The tangents through M to ω intersect ω at T and T'. Find the locus of the centroid of the triangle MTT' a O varies over r.

p8. P, Q and R are the centers of three circles that pass through the same point O. Let A, B and C be the points (other than O) of intersection of the circles. Prove that A, B and C are collinear if and only if O, Q, P and R are in the same circle.

p8 might have a typo, as here (`https://www.oma.org.ar/enunciados/2da2da.htm`) in my source it was incorrect, and I tried correcting it.