Second Round Olympiad 2014

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by sqing, abccsss

1 Let a, b, c be real numbers such that $a+b+c=1$ and $abc>0$. Prove that

$$bc+ca+ab<\frac{\sqrt{abc}}{2}+\frac{1}{4}.$$

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2 Let ABC be an acute triangle such that $\angle BAC \neq 60^\circ$. Let D, E be points such that BD, CE are tangent to the circumcircle of ABC and $BD=CE=BC$ (A is on one side of line BC and D, E are on the other side). Let F, G be intersections of line DE and lines AB, AC. Let M be intersection of CF and BD, and N be intersection of CE and BG. Prove that $AM=AN$.

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3 Let $S=\{1,2,3,\cdots,100\}$. Find the maximum value of integer k, such that there exist k different nonempty subsets of S satisfying the condition: for any two of the k subsets, if their intersection is nonemply, then the minimal element of their intersection is not equal to the maximal element of either of the two subsets.

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4 Let $x_1, x_2, \ldots, x_{2014}$ be integers among which no two are congruent modulo 2014. Let $y_1, y_2, \ldots, y_{2014}$ be integers among which no two are congruent modulo 2014. Prove that one can rearrange $y_1, y_2, \ldots, y_{2014}$ to $z_1, z_2, \ldots, z_{2014}$, so that among

$$x_1+z_1, x_2+z_2, \ldots, x_{2014}+z_{2014}$$

no two are congruent modulo 4028.
