Junior Balkan Team Selection Test 2016

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1 Let rightangled $\triangle ABC$ be given with right angle at vertex C. Let D be foot of altitude from C and let k be circle that touches BD at E, CD at F and circumcircle of $\triangle ABC$ at G. $a.)$ Prove that points A, F and G are collinear. $b.)$ Express radius of circle k in terms of sides of $\triangle ABC$.

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2 Find minimal number of divisors that can number $|2016^m - 36^n|$ have,where m and n are natural numbers.

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3 In two neigbouring cells(dimensions 1×1) of square table 10×10 there is hidden treasure. John needs to guess these cells. In one *move* he can choose some cell of the table and can get information whether there is treasure in it or not. Determine minimal number of *move*'s, with properly strategy, that always allows John to find cells in which is treasure hidden.

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4 Let $a, b, c \in \mathbb{R}^+$, prove that:

$$\frac{2a}{\sqrt{3a+b}} + \frac{2b}{\sqrt{3b+c}} + \frac{2c}{\sqrt{3c+a}} \leq \sqrt{3(a+b+c)}$$
