Greece Team Selection Test 2012

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by gavrilos, silouan

1 Find all triples (p,m,n) satisfying the equation $p^m-n^3=8$ where p is a prime number and m,n are nonnegative integers.

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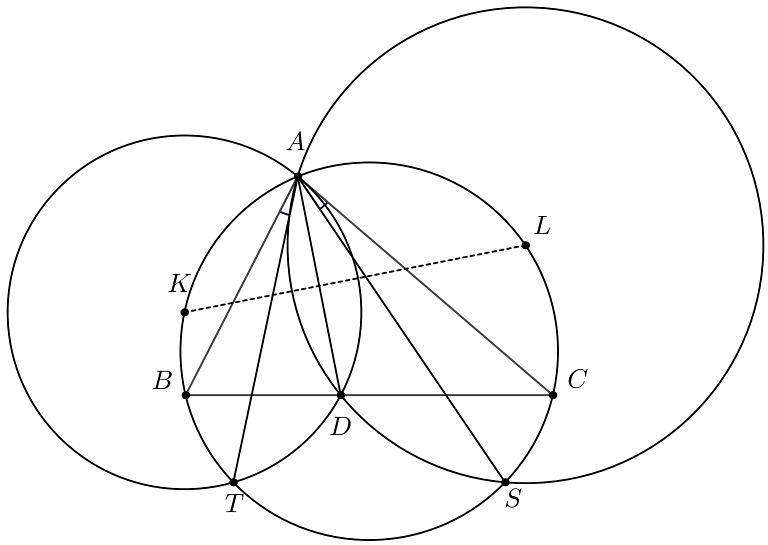
2 Given is an acute triangle ABC $(AB<AC<BC)$,inscribed in circle $c(O,R)$.The perpendicular bisector of the angle bisector AD $(D\in BC)$ intersects c at K,L (K lies on the small arc $\overset{\frown}{AB}$).The circle $c_1(K,KA)$ intersects c at T and the circle $c_2(L,LA)$ intersects c at S.Prove that $\angle BAT=\angle CAS$.



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3 Let a,b,c be positive real numbers satisfying $a+b+c=3$.Prove that $\sum_{sym}\frac{a^2}{(b+c)^3}\geq\frac{3}{8}$

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4 Let $n=3k$ be a positive integer (with $k\geq 2$). An equilateral triangle is divided in n^2 unit equilateral triangles with sides parallel to the initial, forming a grid. We will call "trapezoid" the trapezoid which is formed by three equilateral triangles (one base is equal to one and the other is equal to two). We colour the points of the grid with three colours (red, blue and green) such that each two neighboring points have different colour.
Finally, the colour of a "trapezoid" will be the colour of the midpoint of its big base.
Find the number of all "trapezoids" in the grid (not necessarily disjoint) and determine the number of red, blue and green "trapezoids".
