Greece Team Selection Test 2013

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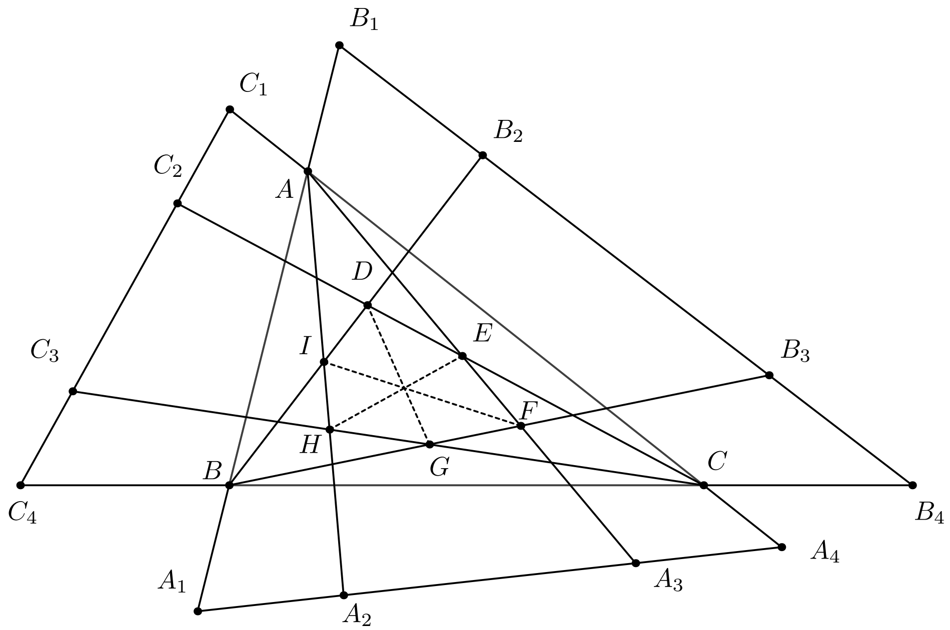
–	Test 1
1	Find all pairs of non-negative integers (m,n) satisfying $\frac{n(n+2)}{4}=m^4+m^2-m+1$
2	Let ABC be a non-isosceles,aqute triangle with $AB<AC$ inscribed in circle $c(O,R)$.The circle $c_1(B,AB)$ crosses AC at K and c at E. KE crosses c at F and BO crosses KE at L and AC at M while AE crosses BF at D.Prove that: i)D,L,M,F are concyclic. ii)B,D,K,M,E are concyclic.
3	Find the largest possible value of M for which $\frac{x}{1+\frac{yz}{x}}+\frac{y}{1+\frac{zx}{y}}+\frac{z}{1+\frac{xy}{z}}\geq M$ for all $x,y,z>0$ with $xy+yz+zx=1$
4	Given are n different concentric circles on the plane.Inside the disk with the smallest radius (strictly inside it),we consider two distinct points A,B.We consider k distinct lines passing through A and m distinct lines passing through B.There is no line passing through both A and B and all the lines passing through k intersect with all the lines passing through B.The intersections do not lie on some of the circles.Determine the maximum and the minimum number of regions formed by the lines and the circles and are inside the circles.
–	Test 2
1	Determine whether the polynomial $P(x)=(x^2-2x+5)(x^2-4x+20)+1$ is irreducible over $\mathbb{Z}[X]$.
2	For the several values of the parameter $m\in \mathbb{N}^*$,find the pairs of integers (a,b) that satisfy the relation $$\frac{[a,m]+[b,m]}{(a+b)m}=\frac{10}{11},$$ and,moreover,on the Cartesian plane Oxy the lie in the square $D=\{(x,y):1\leq x\leq 36,1\leq y\leq 36\}$. [i]Note:[$k,l$] denotes the least common multiple of the positive integers k,l.[/i]
3	Given is a triangle ABC.On the extensions of the side AB we consider points A_1,B_1 such that $AB_1=BA_1$ (with A_1 lying closer to B).On the extensions of the side BC we consider points B_4,C_4 such that $CB_4=BC_4$ (with B_4 lying closer to C).On the extensions of the side AC

we consider points C_1, A_4 such that $AC_1 = CA_4$ (with C_1 lying closer to A).On the segment A_1A_4 we consider points A_2, A_3 such that $A_1A_2 = A_3A_4 = mA_1A_4$ where $0 < m < \frac{1}{2}$.Points B_2, B_3 and C_2, C_3 are defined similarly,on the segments B_1B_4, C_1C_4 respectively.If $D \equiv BB_2 \cap CC_2$, $E \equiv AA_3 \cap CC_2$, $F \equiv AA_3 \cap BB_3$, $G \equiv BB_3 \cap CC_3$, $H \equiv AA_2 \cap CC_3$ and $I \equiv AA_2 \cap BB_2$,prove that the diagonals DG, EH, FI of the hexagon $DEFGHI$ are concurrent.



4 Let n be a positive integer. An equilateral triangle with side n will be denoted by T_n and is divided in n^2 unit equilateral triangles with sides parallel to the initial, forming a grid. We will call ”trapezoid” the trapezoid which is formed by three equilateral triangles (one base is equal to one and the other is equal to two).
Let also m be a positive integer with $m < n$ and suppose that T_n and T_m can be tiled with ”trapezoids”.
Prove that, if from T_n we remove a T_m with the same orientation, then the rest can be tiled with ”trapezoids”.