Greece Team Selection Test 2011

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by silouan

1 Find all prime numbers p, q such that:

$$p^4 + p^3 + p^2 + p = q^2 + q$$

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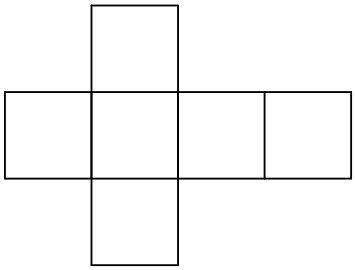
2 What is the maximal number of crosses than can fit in a 10×11 board without overlapping? Is this problem well-known?



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3 Find all functions $f, g : \mathbb{Q} \to \mathbb{Q}$ such that the following two conditions hold:

$$f(g(x) - g(y)) = f(g(x)) - y \quad (1)$$
$$g(f(x) - f(y)) = g(f(x)) - y \quad (2)$$

for all $x, y \in \mathbb{Q}$.

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4 Let $ABCD$ be a cyclic quadrilateral and let K, L, M, N, S, T the midpoints of AB, BC, CD, AD, AC, BD respectively. Prove that the circumcenters of KLS, LMT, MNS, NKT form a cyclic quadrilateral which is similar to $ABCD$.
