Greece Team Selection Test 2009

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1 Suppose that a is an even positive integer and $A=a^n+a^{n-1}+\ldots+a+1, n\in\mathbb{N}^*$ is a perfect square.Prove that $8\mid a$.

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2 Given is a triangle ABC with barycenter G and circumcenter O.The perpendicular bisectors of GA,GB,GC intersect at A_1,B_1,C_1.Show that O is the barycenter of $\triangle A_1B_1C_1$.

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3 Find all triples $(x,y,z)\in\mathbb{R}^3$ such that $x,y,z>3$ and $\frac{(x+2)^2}{y+z-2}+\frac{(y+4)^2}{z+x-4}+\frac{(z+6)^2}{x+y-6}=36$

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4 Given are N points on the plane such that no three of them are collinear,which are coloured red,green and black.We consider all the segments between these points and give to each segment a *"value"* according to the following conditions:
i.If at least one of the endpoints of a segment is black then the segment's *"value"* is 0.
ii. If the endpoints of the segment have the same colour,re or green,then the segment's *"value"* is 1.
iii. If the endpoints of the segment have different colours but none of them is black,then the segment's *"value"* is -1.

Determine the minimum possible sum of the *"values"* of the segments.
