

Austria Beginners' Competition 2012

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by parmenides51

- 1 Let a, b, c and d be four integers such that $7a + 8b = 14c + 28d$.
Prove that the product $a \cdot b$ is always divisible by 14.

 - 2 A postman wants to divide n packages with weights $1, 2, 3, 4, n$ into three groups of exactly the same weight. Can he do this if
 - (a) $n = 2011$?
 - (b) $n = 2012$?

 - 3 Let a and b be two positive real numbers with $a \leq 2b \leq 4a$.
Prove that $4ab \leq 2(a^2 + b^2) \leq 5ab$.

 - 4 A segment AB is given. We erect the equilateral triangles ABC and ADB above and below AB . We denote the midpoints of AC and BC by E and F respectively. Prove that the straight lines DE and DF divide the segment AB into three parts of equal length .
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