Dutch BxMO Team Selection Test 2021

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1 Given is a cyclic quadrilateral $ABCD$ with $|AB| = |BC|$. Point E is on the arc CD where A and B are not on. Let P be the intersection point of BE and CD , let Q be the intersection point of AE and BD . Prove that $PQ \parallel AC$.

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2 Find all triplets (x, y, z) of real numbers for which

$$\begin{cases} x^2 - yz = |y - z| + 1 \\ y^2 - zx = |z - x| + 1 \\ z^2 - xy = |x - y| + 1 \end{cases}$$

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3 Let p be a prime number greater than 2. Patricia wants 7 not-necessarily different numbers from $\{1, 2, ..., p\}$ to the black dots in the figure below, on such a way that the product of three numbers on a line or circle always has the same remainder when divided by p.

`https://cdn.artofproblemsolving.com/attachments/3/1/ef0d63b8ff5341ffc340de0cc75b24c7229e2`
`png`

(a) Suppose Patricia uses the number p at least once. How many times does she have the number p then a minimum sum needed?

(b) Suppose Patricia does not use the number p. In how many ways can she assign numbers? (Two ways are different if there is at least one black one dot different numbers are assigned. The figure is not rotated or mirrored.)

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4 Jesse and Tjeerd are playing a game. Jesse has access to $n \geq 2$ stones. There are two boxes: in the black box there is room for half of the stones (rounded down) and in the white box there is room for half of the stones (rounded up). Jesse and Tjeerd take turns, with Jesse starting. Jesse grabs in his turn, always one new stone, writes a positive real number on the stone and places put him in one of the boxes that isn't full yet. Tjeerd sees all these numbers on the stones in the boxes and on his turn may move any stone from one box to the other box if it is not yet full, but he may also choose to do nothing. The game stops when both boxes are full. If then the total value of the stones in the black box is greater than the total value of the stones in the white box, Jesse wins; otherwise win Tjeerd. For every $n \geq 2$, determine who can definitely win (and give a corresponding winning strategy).

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5 Given is a triangle ABC with the property that $|AB| + |AC| = 3|BC|$. Let T be the point on segment AC such that $|AC| = 4|AT|$. Let K and L be points on the interior of line segments

AB and AC respectively such that $KL \parallel BC$ and KL is tangent to the inscribed circle of $\triangle ABC$. Let S be the intersection of BT and KL. Determine the ratio $\frac{|SL|}{|KL|}$