Berkeley Math Tournament for High School, 2013 - 2021 , T stands for Tiebreaker

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– Geometry Round

2013.1 A rectangle with sides a and b has an area of 24 and a diagonal of length 11. Find the perimeter of this rectangle.

2013.2 Two rays start from a common point and have an angle of 60 degrees. Circle C is drawn with radius 42 such that it is tangent to the two rays. Find the radius of the circle that has radius smaller than circle C and is also tangent to C and the two rays.

2013.3 Given a regular tetrahedron $ABCD$ with center O, find $\sin \angle AOB$.

2013.4 Two cubes A and B have different side lengths, such that the volume of cube A is numerically equal to the surface area of cube B. If the surface area of cube A is numerically equal to six times the side length of cube B, what is the ratio of the surface area of cube A to the volume of cube B?

2013.5 Points A and B are fixed points in the plane such that $AB = 1$. Find the area of the region consisting of all points P such that $\angle APB > 120^o$

2013.6 Let $ABCD$ be a cyclic quadrilateral where $AB = 4$, $BC = 11$, $CD = 8$, and $DA = 5$. If BC and DA intersect at X, find the area of $\triangle XAB$.

2013.7 Let ABC be a triangle with $BC = 5$, $CA = 3$, and $AB = 4$. Variable points P, Q are on segments AB, AC, respectively such that the area of APQ is half of the area of ABC. Let x and y be the lengths of perpendiculars drawn from the midpoint of PQ to sides AB and AC, respectively. Find the range of values of $2y + 3x$.

2013.8 ABC is an isosceles right triangle with right angle B and $AB = 1$. ABC has an incenter at E. The excircle to ABC at side AC is drawn and has center P. Let this excircle be tangent to AB at R. Draw T on the excircle so that RT is the diameter. Extend line BC and draw point D on BC so that DT is perpendicular to RT. Extend AC and let it intersect with DT at G. Let F be the incenter of CDG. Find the area of $\triangle EFP$.

2013.9 Let ABC be a triangle. Points D, E, F are on segments BC, CA, AB, respectively. Suppose that $AF = 10$, $FB = 10$, $BD = 12$, $DC = 17$, $CE = 11$, and $EA = 10$. Suppose that the circumcircles of $\triangle BFD$ and $\triangle CED$ intersect again at X. Find the circumradius of $\triangle EXF$.

2013.10 Let D, E, and F be the points at which the incircle, ω, of $\triangle ABC$ is tangent to BC, CA, and AB, respectively. AD intersects ω again at T. Extend rays TE, TF to hit line BC at E', F', respectively. If $BC = 21$, $CA = 16$, and $AB = 15$, then find $\left|\frac{1}{DE'} - \frac{1}{DF'}\right|$.

2013.P1 Suppose a convex polygon has a perimeter of 1. Prove that it can be covered with a circle of radius $1/4$.

2013.P2 From a point A construct tangents to a circle centered at point O, intersecting the circle at P and Q respectively. Let M be the midpoint of PQ. If K and L are points on circle O such that K, L, and A are collinear, prove $\angle MKO = \angle MLO$.

2014.1 Consider a regular hexagon with an incircle. What is the ratio of the area inside the incircle to the area of the hexagon?

2014.2 Regular hexagon $ABCDEF$ has side length 2 and center O. The point P is defined as the intersection of AC and OB. Find the area of quadrilateral $OPCD$.

2014.3 Consider an isosceles triangle ABC ($AB = BC$). Let D be on BC such that $AD \perp BC$ and O be a circle with diameter BC. Suppose that segment AD intersects circle O at E. If $CA = 2$ what is CE?

2014.4 A cylinder with length ℓ has a radius of 6 meters, and three spheres with radii $3, 4$, and 5 meters are placed inside the cylinder. If the spheres are packed into the cylinder such that ℓ is minimized, determine the length ℓ.

2014.5 In a 100-dimensional hypercube, each edge has length 1. The box contains $2^{100}+1$ hyperspheres with the same radius r. The center of one hypersphere is the center of the hypercube, and it touches all the other spheres. Each of the other hyperspheres is tangent to 100 faces of the hypercube. Thus, the hyperspheres are tightly packed in the hypercube. Find r.

2014.6 Square $ABCD$ has side length 5 and arc BD with center A. E is the midpoint of AB and CE intersects arc BD at F. G is placed onto BC such that FG is perpendicular to BC. What is the length of FG?

2014.7 Consider a parallelogram $ABCD$. E is a point on ray $\overrightarrow{AD}$. BE intersects AC at F and CD at G. If $BF = EG$ and $BC = 3$, find the length of AE

2014.8 Semicircle O has diameter $AB = 12$. Arc $AC = 135^o$. Let D be the midpoint of arc AC. Compute the region bounded by the lines CD and DB and the arc CB.

2014.9 Let ABC be a triangle. Construct points B' and C' such that ACB' and ABC' are equilateral triangles that have no overlap with $\triangle ABC$. Let BB' and CC' intersect at X. If $AX = 3$, $BC = 4$,

and $CX = 5$, find the area of quadrilateral $BCB'C'$.
.

2014.10 Consider 8 points that are a knight's move away from the origin (i.e., the eight points $\{(2, 1)$, $(2, -1)$, $(1, 2)$, $(1, -2)$, $(-1, 2)$, $(-1, -2)$, $(-2, 1)$, $(-2, -1)\}$). Each point has probability $\frac{1}{2}$ of being visible. What is the expected value of the area of the polygon formed by points that are visible? (If exactly $0, 1, 2$ points appear, this area will be zero.)

2014.P1 Let ABC be a triangle. Let r denote the inradius of $\triangle\ ABC$. Let r_a denote the A-exradius of $\triangle\ ABC$. Note that the A-excircle of $\triangle\ ABC$ is the circle that is tangent to segment BC, the extension of ray AB beyond B and the extension of AC beyond C. The A-exradius is the radius of the A-excircle. Define r_b and r_c analogously. Prove that

$$\frac{1}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c}$$

2014.P2 Let ABC be a fixed scalene triangle. Suppose that X, Y are variable points on segments AB, AC, respectively such that $BX = CY$. Prove that the circumcircle of $\triangle\ AXY$ passes through a fixed point other than A.

2015.1 Let ABC be a triangle. The angle bisectors of $\angle ABC$ and $\angle ACB$ intersect at D. If $\angle BAC = 80^o$, what are all possible values for $\angle BDC$?

2015.2 $ABCDEF$ is a regular hexagon. Let R be the overlap between $\triangle\ ACE$ and $\triangle\ BDF$. What is the area of R divided by the area of $ABCDEF$?

2015.3 Let M be on segmentBC of $\triangle\ ABC$ so that $AM = 3$, $BM = 4$, and $CM = 5$. Find the largest possible area of $\triangle\ ABC$.

2015.4 Let $ABCD$ be a rectangle. Circles C_1 and C_2 are externally tangent to each other. Furthermore, C_1 is tangent to AB and AD, and C_2 is tangent to CB and CD. If $AB = 18$ and $BC = 25$, then find the sum of the radii of the circles.

2015.5 Let $A = (1, 0)$, $B = (0, 1)$, and $C = (0, 0)$. There are three distinct points, P, Q, R, such that $\{A, B, C, P\}$, $\{A, B, C, Q\}$, $\{A, B, C, R\}$ are all parallelograms (vertices unordered). Find the area of $\triangle\ PQR$.

2015.6 Let C be the sphere $x^2 + y^2 + (z - 1)^2 = 1$. Point P on C is $(0, 0, 2)$. Let $Q = (14, 5, 0)$. If PQ intersects C again at Q', then find the length PQ'
.

2015.7 Define $A = (1, 0, 0)$, $B = (0, 1, 0)$, and P as the set of all points (x, y, z) such that $x + y + z = 0$. Let P be the point on P such that $d = AP + PB$ is minimized. Find d^2.

2015.8 Suppose that $A = \left(\frac{1}{2}, \sqrt{3}\right)$. Suppose that B, C, D are chosen on the ellipse $x^2 + (y/2)^2 = 1$ such that the area of $ABCD$ is maximized. Assume that A, B, C, D lie on the ellipse going counterclockwise. What are all possible values of B?

2015.9 Let ABC be a triangle. Suppose that a circle with diameter BC intersects segments CA, AB at E, F, respectively. Let D be the midpoint of BC. Suppose that AD intersects EF at X. If $AB = \sqrt{9}$, $AC = \sqrt{10}$, and $BC = \sqrt{11}$, what is $\frac{EX}{XF}$?

2015.10 Let ABC be a triangle with points E, F on CA, AB, respectively. Circle C_1 passes through E, F and is tangent to segment BC at D. Suppose that $AE = AF = EF = 3$, $BF = 1$, and $CE = 2$. What is $\frac{ED^2}{FD^2}$?

2015.P1 Suppose that circles C_1 and C_2 intersect at X and Y . Let A, B be on C_1, C_2, respectively, such that A, X, B lie on a line in that order. Let A, C be on C_1, C_2, respectively, such that A, Y, C lie on a line in that order. Let A', B', C' be another similarly defined triangle with $A \neq A'$. Prove that $BB' = CC'$.

2015.P2 Suppose that fixed circle C_1 with radius $a > 0$ is tangent to the fixed line ℓ at A. Variable circle C_2, with center X, is externally tangent to C_1 at $B \neq A$ and ℓ at C. Prove that the set of all X is a parabola minus a point

2015.T1 Let $ABCD$ be a parallelogram. Suppose that E is on line DC such that C lies on segment ED. Then say lines AE and BD intersect at X and lines CX intersects AB at F. If $AB = 7$,$BC = 13$, and $CE = 91$, then find $\frac{AF}{FB}$.

2015.T2 The unit square $ABCD$ has E as midpoint of AD and a circle of radius r tangent to AB, BC, and CE. Determine r.

2015.T3 The permutohedron of order 3 is the hexagon determined by points $(1, 2, 3)$, $(1, 3, 2)$, $(2, 1, 3)$, $(2, 3, 1)$, $(3, 1, 2)$, and $(3, 2, 1)$. The pyramid determined by these six points and the origin has a unique inscribed sphere of maximal volume. Determine its radius.

2016.1 A $2 \times 4 \times 8$ rectangular prism and a cube have the same volume. What is the difference between their surface areas?

2016.2 Cyclic quadrilateral $ABCD$ has side lengths $AB = 6$, $BC = 7$, $CD = 7$, $DA = 6$. What is the area of $ABCD$?

2016.3 Let S be the set of all non-degenerate triangles with integer sidelengths, such that two of the sides are 20 and 16. Suppose we pick a triangle, at random, from this set. What is the probability that it is acute?

2016.4 ABC is an equilateral triangle, and $ADEF$ is a square. If D lies on side AB and E lies on side BC, what is the ratio of the area of the equilateral triangle to the area of the square?

2016.5 Convex pentagon $ABCDE$ has the property that $\angle ADB = 20^o$, $\angle BEC = 16^o$, $\angle CAD = 3^o$,and $\angle DBE = 12^o$. What is the measure of $\angle ECA$?

2016.6 Triangle ABC has sidelengths $AB = 13$, $AC = 14$, and $BC = 15$ and centroid G. What is the area of the triangle with sidelengths AG, BG, and CG

2016.7 Let ABC be a right triangle with $AB = BC = 2$. Construct point D such that $\angle DAC = 30^o$ and $\angle DCA = 60^o$, and $\angle BCD > 90^o$. Compute the area of triangle BCD.

2016.8 A regular unit 7-simplex is a polytope in 7-dimensional space with 8 vertices that are all exactly a distance of 1 apart. (It is the 7-dimensional analogue to the triangle and the tetrahedron.) In this 7-dimensional space, there exists a point that is equidistant from all 8 vertices, at a distance d. Determine d.

2016.9 Given right triangle ABC with right angle at C, construct three external squares $ABDE$, $BCFG$, and $ACHI$. If $DG = 19$ and $EI = 22$, compute the length of FH.

2016.10 Triangle ABC has side lengths $AB = 5$, $BC = 9$, and $AC = 6$. Define the incircle of ABC to be C_1. Then, define C_i for $i > 1$ to be externally tangent to C_{i-1} and tangent to AB and BC. Compute the sum of the areas of all circles C_n.

2017.1 What is the largest n such that there exists a non-degenerate convex n-gon such that each of its angles are an integer number of degrees, and are all distinct?

2017.2 Let S be the set of points A in the xy-plane such that the four points A, $(2, 3)$, $(-1, 0)$, and $(0, 6)$ form the vertices of a parallelogram. Let P be the convex polygon whose vertices are the points in S. What is the area of P?

2017.3 Let $ABCDEF$ be a regular hexagon with side length 1. Now, construct square $AGDQ$. What is the area of the region inside the hexagon and not the square?

2017.4 How many lattice points (v, w, x, y, z) does a 5-sphere centered on the origin, with radius 3, contain on its surface or in its interior?

2017.5 Suppose the side lengths of triangle ABC are the roots of polynomial $x^3 - 27x^2 + 222x - 540$. What is the product of its inradius and circumradius?

2017.6 Given a cube with side length 1, we perform six cuts as follows: one cut parallel to the xy-plane, two cuts parallel to the yz-plane, and three cuts parallel to the xz-plane, where the cuts are made uniformly independent of each other. What is the expected value of the volume of the largest piece?

2017.7 Determine the maximal area triangle such that all of its vertices satisfy $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

2017.8 Given a circle of radius 25, consider the set of triangles with area at least 768. What is the area of the intersection of all the triangles in this set?

2017.9 Let $\triangle ABC$ be a triangle. Let D be the point on BC such that DA is tangent to the circumcircle of ABC. Let E be the point on the circumcircle of ABC such that DE is tangent to the circumcircle of ABC, but $E \neq A$. Let F be the intersection of AE and BC. Given that $BF/FC = 4/5$, find the maximum possible value for $\sin \angle ACB$/

2017.10 Colorado and Wyoming are both defined to be 4 degrees tall in latitude and 7 degree wide in longitude. In particular, Colorado is defined to be at $37^o N$ to $41^o N$, and $102^o 03' W$ to $109^o 03' W$, whereas Wyoming is defined to be $41^o N$ to $45^o N$, and $104^o 03' W$ to $111^o 03' W$. Assuming Earth is a perfect sphere with radius R, what is the ratio of the areas of Wyoming to Colorado, in terms of R?

2018.1 A cube has side length 5. Let S be its surface area and V its volume. Find $\frac{S^3}{V^2}$.

2018.2 A 1 by 1 square $ABCD$ is inscribed in the circle m. Circle n has radius 1 and is centered around A. Let S be the set of points inside of m but outside of n. What is the area of S?

2018.3 If A is the area of a triangle with perimeter 1, what is the largest possible value of A^2?

2018.4 There are six lines in the plane. No two of them are parallel and no point lies on more than three lines. What is the minimum possible number of points that lie on at least two lines?

2018.5 A point is picked uniformly at random inside of a square. Four segments are then drawn in connecting the point to each of the vertices of the square, cutting the square into four triangles. What is the probability that at least two of the resulting triangles are obtuse?

2018.6 A triangle T has all integer side lengths and at most one of its side lengths is greater than ten. What is the largest possible area of T ?

2018.7 A line in the xy-plane has positive slope, passes through the point $(x, y) = (0, 29)$, and lies tangent to the ellipse defined by $\frac{x^2}{100} + \frac{y^2}{400} = 1$. What is the slope of the line?

2018.8 What is the largest possible area of a triangle with largest side length 39 and inradius 10?

2018.9 What is the least integer a greater than 14 so that the triangle with side lengths $a - 1$, a, and $a + 1$ has integer area?

2018.10 A plane cuts a sphere of radius 1 into two pieces, one of which has three times the surface area of the other. What is the area of the disk that the sphere cuts out of the plane?

2018.T1 Line segment $\overline{AE}$ of length 17 bisects $\overline{DB}$ at a point C. If $\overline{AB} = 5$, $\overline{BC} = 6$ and $\angle BAC = 78^o$ degrees, calculate $\angle CDE$.

2018.T2 Points A, B, C are chosen on the boundary of a circle with center O so that $\angle BAC$ encloses an arc of 120 degrees. Let D be chosen on $\overline{BA}$ so that $\angle AOD$ is a right angle. Extend $\overline{CD}$ so that it intersects with O again at point P. What is the measure of the arc, in degrees, that is enclosed by $\angle ACP$? Please use the tan^{-1} function to express your answer.

2018.T3 Consider a regular polygon with 2^n sides, for $n \ge 2$, inscribed in a circle of radius 1. Denote the area of this polygon by A_n. Compute $\prod_{i=2}^{\infty} \frac{A_i}{A_{i+1}}$

2019.1 Consider the figure (attached), where every small triangle is equilateral with side length 1. Compute the area of the polygon $AEKS$.
`https://cdn.artofproblemsolving.com/attachments/c/7/671748fe0fce7b8f89532ca66011d119f9b7a`
`png`
Posted for the link in the related post collection (`https://artofproblemsolving.com/community/c2503497_2019_bmt_spring`), with the figure

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2019.2 A set of points in the plane is called *full* if every triple of points in the set are the vertices of a non-obtuse triangle. What is the largest size of a full set?

2019.3 Let $ABCD$ be a parallelogram with $BC = 17$. Let M be the midpoint of $\overline{BC}$ and let N be the point such that $DANM$ is a parallelogram. What is the length of segment $\overline{NC}$?

2019.4 The area of right triangle ABC is 4, and the length of hypotenuse AB is 12. Compute the perimeter of $\triangle ABC$.

2019.5 Find the area of the set of all points z in the complex plane that satisfy $|z - 3i| + |z - 4| \le 5\sqrt{2}$.

2019.6 Let $\triangle ABE$ be a triangle with $\frac{AB}{3} = \frac{BE}{4} = \frac{EA}{5}$. Let $D \neq A$ be on line $\overline{AE}$ such that $AE = ED$ and D is closer to E than to A. Moreover, let C be a point such that $BCDE$ is a parallelogram. Furthermore, let M be on line $\overline{CD}$ such that $\overline{AM}$ bisects $\angle BAE$, and let P be the intersection of $\overline{AM}$ and $\overline{BE}$. Compute the ratio of PM to the perimeter of $\triangle ABE$.

2019.7 Points A, B, C, D are vertices of an isosceles trapezoid, with $\overline{AB}$ parallel to $\overline{CD}$, $AB = 1$, $CD = 2$, and $BC = 1$. Point E is chosen uniformly and at random on $\overline{CD}$, and let point F be the point on $\overline{CD}$ such that $EC = FD$. Let G denote the intersection of $\overline{AE}$ and $\overline{BF}$, not necessarily in the trapezoid. What is the probability that $\angle AGB > 30°$?

2019.8 Let $\triangle ABC$ be a triangle with $AB = 13$, $BC = 14$, and $CA = 15$. Let G denote the centroid of $\triangle ABC$, and let G_A denote the image of G under a reflection across $\overline{BC}$, with G_B the image of G under a reflection across $\overline{AC}$, and G_C the image of G under a reflection across $\overline{AB}$. Let O_G be the circumcenter of $\triangle G_AG_BG_C$ and let X be the intersection of $\overline{AO_G}$ with $\overline{BC}$. Let Y denote the intersection of $\overline{AG}$ with $\overline{BC}$. Compute XY.

2019.9 Let $ABCD$ be a tetrahedron with $\angle ABC = \angle ABD = \angle CBD = 90^\circ$ and $AB = BC$. Let E, F, G be points on $\overline{AD}$, BD, and $\overline{CD}$, respectively, such that each of the quadrilaterals $AEFB$, $BFGC$, and $CGEA$ have an inscribed circle. Let r be the smallest real number such that $\frac{[\triangle EFG]}{[\triangle ABC]} \le r$ for all such configurations A, B, C, D, E, F, G. If r can be expressed as $\frac{\sqrt{a - b\sqrt{c}}}{d}$ where a, b, c, d are positive integers with $\gcd(a, b)$ squarefree and c squarefree, find $a + b + c + d$.

Note: Here, $[P]$ denotes the area of polygon P. (This wasn't in the original test; instead they used the notation area(P), which is clear but frankly cumbersome. :P)

2019.10 A *3-4-5 point* of a triangle ABC is a point P such that the ratio $AP : BP : CP$ is equivalent to the ratio $3 : 4 : 5$. If $\triangle ABC$ is isosceles with base $BC = 12$ and $\triangle ABC$ has exactly one $3-4-5$ point, compute the area of $\triangle ABC$.

2019.T1 We inscribe a circle ω in equilateral triangle ABC with radius 1. What is the area of the region inside the triangle but outside the circle?

2019.T2 Define the *inverse* of triangle ABC with respect to a point O in the following way: construct the circumcircle of ABC and construct lines AO, BO, and CO. Let A' be the other intersection of AO and the circumcircle (if AO is tangent, then let $A' = A$). Similarly define B' and C'. Then $A'B'C'$ is the inverse of ABC with respect to O. Compute the area of the inverse of the triangle given in the plane by $A(-6, -21)$, $B(-23, 10)$, $C(16, 23)$ with respect to $O(1, 3)$.

2019.T3 We say that a quadrilateral Q is *tangential* if a circle can be inscribed into it, i.e. there exists a circle C that does not meet the vertices of Q, such that it meets each edge at exactly one point. Let N be the number of ways to choose four distinct integers out of $\{1, ..., 24\}$ so that they form the side lengths of a tangential quadrilateral. Find the largest prime factor of N.

2020.1 A Yule log is shaped like a right cylinder with height 10 and diameter 5. Freya cuts it parallel to its bases into 9 right cylindrical slices. After Freya cut it, the combined surface area of the slices of the Yule log increased by $a\pi$. Compute a.

2020.2 Let O be a circle with diameter $AB = 2$. Circles O_1 and O_2 have centers on $\overline{AB}$ such that O is tangent to O_1 at A and to O_2 at B, and O_1 and O_2 are externally tangent to each other. The minimum possible value of the sum of the areas of O_1 and O_2 can be written in the form $\frac{m\pi}{n}$

where m and n are relatively prime positive integers. Compute $m + n$.

2020.3 Right triangular prism $ABCDEF$ with triangular faces $\triangle$ ABC and $\triangle$ DEF and edges $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ has $\angle ABC = 90^o$ and $\angle EAB = \angle CAB = 60^o$. Given that $AE = 2$, the volume of $ABCDEF$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.
https://cdn.artofproblemsolving.com/attachments/4/7/25fbe2ce2df50270b48cc503a8af4e0c01302
png

2020.4 Alice is standing on the circumference of a large circular room of radius 10. There is a circular pillar in the center of the room of radius 5 that blocks Alice's view. The total area in the room Alice can see can be expressed in the form $\frac{m\pi}{n} + p\sqrt{q}$, where m and n are relatively prime positive integers and p and q are integers such that q is square-free. Compute $m + n + p + q$. (Note that the pillar is not included in the total area of the room.)
https://cdn.artofproblemsolving.com/attachments/1/9/a744291a61df286735d63d8eb09e25d462785
png

2020.5 Let $A_1 = (0, 0)$, $B_1 = (1, 0)$, $C_1 = (1, 1)$, $D_1 = (0, 1)$. For all $i > 1$, we recursively define

$$A_i = \frac{1}{2020}(A_{i-1} + 2019B_{i-1}), B_i = \frac{1}{2020}(B_{i-1} + 2019C_{i-1})$$

$$C_i = \frac{1}{2020}(C_{i-1} + 2019D_{i-1}), D_i = \frac{1}{2020}(D_{i-1} + 2019A_{i-1})$$

where all operations are done coordinate-wise.
https://cdn.artofproblemsolving.com/attachments/8/7/9b6161656ed2bc70510286dc8cb75cc5bde6c
png
If $[A_iB_iC_iD_i]$ denotes the area of $A_iB_iC_iD_i$, there are positive integers a, b, and c such that $\sum_{i=1}^{\infty}[A_iB_iC_iD_i] = \frac{a^2b}{c}$, where b is square-free and c is as small as possible. Compute the value of $a + b + c$

2020.6 A tetrahedron has four congruent faces, each of which is a triangle with side lengths 6, 5, and 5. If the volume of the tetrahedron is V , compute V^2
.

2020.7 Circle Γ has radius 10, center O, and diameter $\overline{AB}$. Point C lies on Γ such that $AC = 12$. Let P be the circumcenter of $\triangle$ AOC. Line AP intersects Γ at Q, where Q is different from A. Then the value of $\frac{AP}{AQ}$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

2020.8 Let triangle $\triangle$ ABC have $AB = 17$, $BC = 14$, $CA = 12$. Let M_A, M_B, M_C be midpoints of $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ respectively. Let the angle bisectors of A, B, and C intersect $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ at P, Q, and R, respectively. Reflect M_A about $\overline{AP}$, M_B about $\overline{BQ}$, and M_C about $\overline{CR}$ to obtain M'_A,

M'_B, M'_C, respectively. The lines AM'_A, BM'_B, and CM'_C will then intersect $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ at D, E, and F, respectively. Given that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ concur at a point K inside the triangle, in simplest form, the ratio $[KAB]:[KBC]:[KCA]$ can be written in the form $p:q:r$, where p, q and r are relatively prime positive integers and $[XYZ]$ denotes the area of $\triangle XYZ$. Compute $p+q+r$.

2020.9 The Fibonacci numbers F_n are defined as $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n > 2$. Let A be the minimum area of a (possibly degenerate) convex polygon with 2020 sides, whose side lengths are the first 2020 Fibonacci numbers F_1, F_2, ... , F_{2020} (in any order). A degenerate convex polygon is a polygon where all angles are $\le 180^o$. If A can be expressed in the form

$$\frac{\sqrt{(F_a - b)^2 - c}}{d}$$

, where a, b, c and d are positive integers, compute the minimal possible value of $a+b+c+d$.

2020.10 Let E be an ellipse where the length of the major axis is 26, the length of the minor axis is 24, and the foci are at points R and S. Let A and B be points on the ellipse such that $RASB$ forms a non-degenerate quadrilateral, lines RA and SB intersect at P with segment PR containing A, and lines RB and AS intersect at Q with segment QR containing B. Given that $RA = AS$, $AP = 26$, the perimeter of the non-degenerate quadrilateral $RPSQ$ is $m + \sqrt{n}$, where m and n are integers. Compute $m + n$.

2020.T1 Given a regular hexagon, a circle is drawn circumscribing it and another circle is drawn inscribing it. The ratio of the area of the larger circle to the area of the smaller circle can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

2020.T2 Quadrilateral $ABCD$ is cyclic with $AB = CD = 6$. Given that $AC = BD = 8$ and $AD + 3 = BC$, the area of $ABCD$ can be written in the form $\frac{p\sqrt{q}}{r}$, where p, q, and r are positive integers such that p and r are relatively prime and that q is square-free. Compute $p + q + r$.

2020.T3 In unit cube $ABCDEFGH$ (with faces $ABCD$, $EFGH$ and connecting vertices labeled so that $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, $\overline{DH}$ are edges of the cube), L is the midpoint of GH. The area of $\triangle\, CAL$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.

2021.1 Shreyas has a rectangular piece of paper $ABCD$ such that $AB = 20$ and $AD = 21$. Given that Shreyas can make exactly one straight-line cut to split the paper into two pieces, compute the maximum total perimeter of the two pieces

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2021.2 Compute the area of the smallest triangle which can contain six congruent, non-overlapping unit circles.

2021.3 In quadrilateral $ABCD,$ suppose that $\overline{CD}$ is perpendicular to $\overline{BC}$ and $\overline{DA}$. Point E is chosen on segment $\overline{CD}$ such that $\angle AED = \angle BEC$. If $AB = 6$, $AD = 7$, and $\angle ABC = 120^o$, compute $AE + EB$.

2021.4 An equilateral polygon has unit side length and alternating interior angle measures of 15^o and 300^o. Compute the area of this polygon.

2021.5 Let circles ω_1 and ω_2 intersect at P and Q. Let the line externally tangent to both circles that is closer to Q touch ω_1 at A and ω_2 at B. Let point T lie on segmentPQ such that $\angle ATB = 90^o$. Given that $AT = 6$, $BT = 8$, and $PT = 4$, compute PQ.

2021.6 Consider 27 unit-cubes assembled into one $3 \times 3 \times 3$ cube. Let A and B be two opposite corners of this large cube. Remove the one unit-cube not visible from the exterior, along with all six unit-cubes in the center of each face. Compute the minimum distance an ant has to walk along the surface of the modified cube to get from A to B.
https://cdn.artofproblemsolving.com/attachments/0/5/d3aa802eae40cfe717088445daabd5e71946
png

2021.7 The line ℓ passes through vertex$B$ and the interior of regular hexagon $ABCDEF$. If the distances from ℓ to the vertices A and C are 7 and 4, respectively, compute the area of hexagon $ABCDEF$.

2021.8 Let $\triangle ABC$ be a triangle with $AB = 15$, $AC = 13$, $BC = 14$, and circumcenter O. Let ℓ be the line through A perpendicular to segment BC. Let the circumcircle of $\triangle AOB$ and the circumcircle of $\triangle AOC$ intersect ℓ at points X and Y (other than A), respectively. Compute the length of $\overline{XY}$
.

2021.9 Let $ABCD$ be a convex quadrilateral such that $\triangle ABC$ is equilateral. Let P be a point inside the quadrilateral such that $\triangle APD$ is equilateral and $\angle PCD = 30^o$. Given that $CP = 2$ and $CD = 3$, compute the area of the triangle formed by P, the midpoint of segment $\overline{BC}$, and the midpoint of segment $\overline{AB}$.

2021.10 Consider $\triangle ABC$ such that $CA + AB = 3BC$. Let the incircle ω touch segments $\overline{CA}$ and $\overline{AB}$ at E and F, respectively, and define P and Q such that segments $\overline{PE}$ and $\overline{QF}$ are diameters of ω. Define the function D of a point K to be the sum of the distances from K to P and Q (i.e. $D(K) = KP + KQ$). Let W, X, Y , and Z be points chosen on lines $\overleftrightarrow{BC}$, $\overleftrightarrow{CE}$, $\overleftrightarrow{EF}$, and $\overleftrightarrow{FB}$, respectively. Given that $BC = \sqrt{133}$ and the inradius of $\triangle ABC$ is $\sqrt{14}$, compute the minimum value of $D(W) + D(X) + D(Y) + D(Z)$.

2021.T1 Regular hexagon $NOSAME$ with side length 1 and square $UDON$ are drawn in the plane such that $UDON$ lies outside of $NOSAME$. Compute $[SAND] + [SEND]$, the sum of the areas of quadrilaterals $SAND$ and $SEND$.

2021.T2 Let $\triangle A_0B_0C_0$ be an equilateral triangle with area 1, and let A_1, B_1, C_1 be the midpoints of $\overline{A_0B_0}$, $\overline{B_0C_0}$, and $\overline{C_0A_0}$, respectively. Furthermore, set A_2, B_2, C_2 as the midpoints of segments $\overline{A_0A_1}$, $\overline{B_0B_1}$, and $\overline{C_0C_1}$ respectively. For $n \geq 1$, A_{2n+1} is recursively defined as the midpoint of $A_{2n}A_{2n-1}$, and A_{2n+2} is recursively defined as the midpoint of $\overline{A_{2n+1}A_{2n-1}}$. Recursively define B_n and C_n the same way. Compute the value of $\lim_{n\to\infty}[A_nB_nC_n]$, where $[A_nB_nC_n]$ denotes the area of triangle $\triangle A_nB_nC_n$.

2021.T3 Right triangle $\triangle ABC$ with its right angle at B has angle bisector $\overline{AD}$ with D on $\overline{BC}$, as well as altitude $\overline{BE}$ with E on $\overline{AC}$. If $\overline{DE} \perp \overline{BC}$ and $AB = 1$, compute AC.
