8th IGO

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– Elementary

1 With putting the four shapes drawn in the following figure together make a shape with at least two reflection symmetries.
`https://cdn.artofproblemsolving.com/attachments/6/0/8ace983d3d9b5c7f93b03c505430e1d2d189f png`

Proposed by Mahdi Etesamifard - Iran

2 Points K, L, M, N lie on the sides AB, BC, CD, DA of a square $ABCD$, respectively, such that the area of $KLMN$ is equal to one half of the area of $ABCD$. Prove that some diagonal of $KLMN$ is parallel to some side of $ABCD$.

Proposed by Josef Tkadlec - Czech Republic

3 As shown in the following figure, a heart is a shape consist of three semicircles with diameters AB, BC and AC such that B is midpoint of the segment AC. A heart ω is given. Call a pair (P, P') bisector if P and P' lie on ω and bisect its perimeter. Let (P, P') and (Q, Q') be bisector pairs. Tangents at points $P, P', Q,$ and Q' to ω construct a convex quadrilateral $XYZT$. If the quadrilateral $XYZT$ is inscribed in a circle, find the angle between lines PP' and QQ'.
`https://cdn.artofproblemsolving.com/attachments/3/c/8216889594bbb504372d8cddfac73b9f56e74 png`

Proposed by Mahdi Etesamifard - Iran

4 In isosceles trapezoid $ABCD$ ($AB \parallel CD$) points E and F lie on the segment CD in such a way that D, E, F and C are in that order and $DE = CF$. Let X and Y be the reflection of E and C with respect to AD and AF. Prove that circumcircles of triangles ADF and BXY are concentric.

Proposed by Iman Maghsoudi - Iran

5 Let $A_1, A_2, ..., A_{2021}$ be 2021 points on the plane, no three collinear and

$$\angle A_1A_2A_3 + \angle A_2A_3A_4 + ... + \angle A_{2021}A_1A_2 = 360^o,$$

in which by the angle $\angle A_{i-1}A_iA_{i+1}$ we mean the one which is less than 180^o (assume that $A_{2022} = A_1$ and $A_0 = A_{2021}$). Prove that some of these angles will add up to 90^o.

Proposed by Morteza Saghafian - Iran

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– Intermediate

1 Let ABC be a triangle with $AB = AC$. Let H be the orthocenter of ABC. Point E is the midpoint of AC and point D lies on the side BC such that $3CD = BC$. Prove that $BE \perp HD$.

Proposed by Tran Quang Hung - Vietnam

2 Let $ABCD$ be a parallelogram. Points E, F lie on the sides AB, CD respectively, such that $\angle EDC = \angle FBC$ and $\angle ECD = \angle FAD$. Prove that $AB \geq 2BC$.

Proposed by Pouria Mahmoudkhan Shirazi - Iran

3 Given a convex quadrilateral $ABCD$ with $AB = BC$and $\angle ABD = \angle BCD = 90$.Let point E be the intersection of diagonals AC and BD. Point F lies on the side AD such that $\frac{AF}{FD} = \frac{CE}{EA}$.. Circle ω with diameter DF and the circumcircle of triangle ABF intersect for the second time at point K. Point L is the second intersection of EF and ω. Prove that the line KL passes through the midpoint of CE.
Proposed by Mahdi Etesamifard and Amir Parsa Hosseini - Iran

4 Let ABC be a scalene acute-angled triangle with its incenter I and circumcircle Γ.
Line AI intersects Γ for the second time at M. Let N be the midpoint of BC and T be the point on Γ such that $IN \perp MT$. Finally, let P and Q be the intersection points of TB and TC, respectively, with the line perpendicular to AI at I. Show that $PB = CQ$.
Proposed by Patrik Bak - Slovakia

5 Consider a convex pentagon $ABCDE$ and a variable point X on its side CD.
Suppose that points K, L lie on the segment AX such that $AB = BK$ and $AE = EL$ and that the circumcircles of triangles CXK and DXL intersect for the second time at Y . As X varies, prove that all such lines XY pass through a fixed point, or they are all parallel.
Proposed by Josef Tkadlec - Czech Republic

– Advanced

1 Acute-angled triangle ABC with circumcircle ω is given. Let D be the midpoint of AC, E be the foot of altitude from A to BC, and F be the intersection point of AB and DE. Point H lies on the arc BC of ω (the one that does not contain A) such that $\angle BHE = \angle ABC$. Prove that $\angle BHF = 90°$.

2 Two circles Γ_1 and Γ_2 meet at two distinct points A and B. A line passing through A meets Γ_1 and Γ_2 again at C and D respectively, such that A lies between C and D. The tangent at A to Γ_2 meets Γ_1 again at E. Let F be a point on Γ_2 such that F and A lie on different sides of BD, and $2\angle AFC = \angle ABC$. Prove that the tangent at F to Γ_2, and lines BD and CE are concurrent.

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3 Consider a triangle ABC with altitudes AD, BE, and CF, and orthocenter H. Let the perpendicular line from H to EF intersects EF, AB and AC at P, T and L, respectively. Point K lies on the side BC such that $BD = KC$. Let ω be a circle that passes through H and P, that is tangent to AH. Prove that circumcircle of triangle ATL and ω are tangent, and KH passes through the tangency point.

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4 2021 points on the plane in the convex position, no three collinear and no four concyclic, are given. Prove that there exist two of them such that every circle passing through these two points contains at least 673 of the other points in its interior.
(A finite set of points on the plane are in convex position if the points are the vertices of a convex polygon.)

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5 Given a triangle ABC with incenter I. The incircle of triangle ABC is tangent to BC at D. Let P and Q be points on the side BC such that $\angle PAB = \angle BCA$ and $\angle QAC = \angle ABC$, respectively. Let K and L be the incenter of triangles ABP and ACQ, respectively. Prove that AD is the Euler line of triangle IKL.

Proposed by Le Viet An, Vietnam

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