2020 Junior Balkan Team Selection Tests-Serbia

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1 Prove that for positive real numbers a, b, c the following inequality holds:

$$\frac{a}{9bc+1}+\frac{b}{9ca+1}+\frac{c}{9ab+1}\geq\frac{a+b+c}{1+(a+b+c)^2}$$

When does equality occur?

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2 Solve the following equation in natural numbers:

$$x^2=2^y+2021^z$$

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3 Two players play the following game: alternatively they write numbers 1 or 0 in the vertices of an n-gon.
First player starts the game and wins if after any of his moves there exists a triangle, whose vertices are three consecutive vertices of the n-gon, such that the sum of numbers in it's vertices is divisible by 3.
Second player wins if he prevents this.
Determine which player has a winning strategy if:
a) $n=2019$
b) $n=2020$
c) $n=2021$

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4 On sides AB and AC of an acute triangle ΔABC, with orthocenter H and circumcenter O, are given points P and Q respectively such that $APHQ$ is a parallelogram. Prove the following equality:

$$\frac{PB\cdot PQ}{QA\cdot QO}=2$$
