Turkey Junior National Olympiad 2021

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1 Find all (m, n) positive integer pairs such that both $\frac{3n^2}{m}$ and $\sqrt{n^2+m}$ are integers.

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2 We are numbering the rows and columns of a 29x29 chess table with numbers $1, 2, ..., 29$ in order (Top row is numbered with 1 and first columns is numbered with 1 as well). We choose some of the squares in this chess table and for every selected square, we know that there exist at most one square having a row number greater than or equal to this selected square's row number and a column number greater than or equal to this selected square's column number. How many squares can we choose at most?

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3 Let x, y, z be real numbers such that

$$x+y+z=2, \quad xy+yz+zx=1$$

Find the maximum possible value of $x-y$.

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4 Let X be a point on the segment $[BC]$ of an equilateral triangle ABC and let Y and Z be points on the rays $[BA$ and $[CA$ such that the lines AX, BZ, CY are parallel. If the intersection of XY and AC is M and the intersection of XZ and AB is N, prove that MN is tangent to the incenter of ABC.
