Berkley mini Math Tournament Fall 2012

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by parmenides51

Team Round **p1.** Ed, Fred and George are playing on a see-saw that is slightly off center. When Ed sits on the left side and George, who weighs 100 pounds, on the right side, they are perfectly balanced. Similarly, if Fred, who weighs 400 pounds, sits on the left and Ed sits on the right, they are also perfectly balanced. Assuming the see-saw has negligible weight, what is the weight of Ed, in pounds?

p2. How many digits does the product $2^{42} \cdot 5^{38}$ have?

p3. Square $ABCD$ has equilateral triangles drawn external to each side, as pictured. If each triangle is folded upwards to meet at a point E, then a square pyramid can be made. If the center of square $ABCD$ is O, what is the measure of $\angle OEA$?
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`png`

p4. How many solutions (x, y) in the positive integers are there to $3x + 7y = 1337$?

p5. A trapezoid with height 12 has legs of length 20 and 15 and a larger base of length 42. What are the possible lengths of the other base?

p6. Let $f(x) = 6x + 7$ and $g(x) = 7x + 6$. Find the value of a such that $g^{-1}(f^{-1}(g(f(a)))) = 1$.

p7. Billy and Cindy want to meet at their favorite restaurant, and they have made plans to do so sometime between $1 : 00$ and $2 : 00$ this Sunday. Unfortunately, they didn't decide on an exact time, so they both decide to arrive at a random time between $1 : 00$ and $2 : 00$. Silly Billy is impatient, though, and if he has to wait for Cindy, he will leave after 15 minutes. Cindy, on the other hand, will happily wait for Billy from whenever she arrives until $2 : 00$. What is the probability that Billy and Cindy will be able to dine together?

p8. As pictured, lines are drawn from the vertices of a unit square to an opposite trisection point. If each triangle has legs with ratio $3 : 1$, what is the area of the shaded region?
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p9. For any positive integer n, let $f_1(n)$ denote the sum of the squares of the digits of n. For $k \ge 2$, let $f_k(n) = f_{k-1}(f_1(n))$. Then, $f_1(5) = 25$ and $f_3(5) = f_2(25) = 85$. Find $f_{2012}(15)$.

p10. Given that 2012022012 has 8 distinct prime factors, find its largest prime factor.

PS. You had better use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

Ind. Round **p1.** What is the slope of the line perpendicular to the the graph $\frac{x}{4} + \frac{y}{9} = 1$ at $(0, 9)$?

p2. A boy is standing in the middle of a very very long staircase and he has two pogo sticks. One pogo stick allows him to jump 220 steps up the staircase. The second pogo stick allows him to jump 125 steps down the staircase. What is the smallest positive number of steps that he can reach from his original position by a series of jumps?

p3. If you roll three fair six-sided dice, what is the probability that the product of the results will be a multiple of 3?

p4. Right triangle ABC has squares $ABXY$ and $ACWZ$ drawn externally to its legs and a semicircle drawn externally to its hypotenuse BC. If the area of the semicircle is 18π and the area of triangle ABC is 30, what is the sum of the areas of squares $ABXY$ and $ACWZ$?
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p5. You have a bag containing 3 types of pens: red, green, and blue. 30% of the pens are red pens, and 20% are green pens. If, after you add 10 blue pens, 60% of the pens are blue pens, how many green pens did you start with?

p6. Canada gained partial independence from the United Kingdom in 1867, beginning its long role as the headgear of the United States. It gained its full independence in 1982. What is the last digit of 1867^{1982}?

p7. Bacon, Meat, and Tomato are dealing with paperwork. Bacon can fill out 5 forms in 3 minutes, Meat can fill out 7 forms in 5 minutes, and Tomato can staple 3 forms in 1 minute. A form must be filled out and stapled together (in either order) to complete it. How long will it take them to complete 105 forms?

p8. Nice numbers are defined to be 7-digit palindromes that have no 3 identical digits (e.g.,

1234321 or 5610165 but not 7427247). A pretty number is a nice number with a 7 in its decimal representation (e.g., 3781873). What is the 7^{th} pretty number?

p9. Let O be the center of a semicircle with diameter AD and area 2π. Given square $ABCD$ drawn externally to the semicircle, construct a new circle with center B and radius BO. If we extend BC, this new circle intersects BC at P. What is the length of CP?
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p10. Derek has 10 American coins in his pocket, summing to a total of 53 cents. If he randomly grabs 3 coins from his pocket, what is the probability that they're all different?

p11. What is the sum of the whole numbers between $6\sqrt{10}$ and 7π ?

p12. What is the volume of a cylinder whose radius is equal to its height and whose surface area is numerically equal to its volume?

p13. 15 people, including Luke and Matt, attend a Berkeley Math meeting. If Luke and Matt sit next to each other, a fight will break out. If they sit around a circular table, all positions equally likely, what is the probability that a fight doesn't break out?

p14. A non-degenerate square has sides of length s, and a circle has radius r. Let the area of the square be equal to that of the circle. If we have a rectangle with sides of lengths r, s, and its area has an integer value, what is the smallest possible value for s?

p15. How many ways can you arrange the letters of the word "$BERKELEY$" such that no two E's are next to each other?

p16. Kim, who has a tragic allergy to cake, is having a birthday party. She invites 12 people but isn't sure if 11 or 12 will show up. However, she needs to cut the cake before the party starts. What is the least number of slices (not necessarily of equal size) that she will need to cut to ensure that the cake can be equally split among either 11 or 12 guests with no excess?

p17. Tom has 2012 blue cards, 2012 red cards, and 2012 boxes. He distributes the cards in such a way such that each box has at least 1 card. Sam chooses a box randomly, then chooses a card randomly. Suppose that Tom arranges the cards such that the probability of Sam choosing a blue card is maximized. What is this maximum probability?

p18. Allison wants to bake a pie, so she goes to the discount market with a handful of dollar bills. The discount market sells different fruit each for some integer number of cents and does not add tax to purchases. She buys 22 apples and 7 boxes of blueberries, using all of her dollar bills. When she arrives back home, she realizes she needs more fruit, though, so she grabs her checkbook and heads back to the market. This time, she buys 31 apples and 4 boxes of blueberries, for a total of 60 cents more than her last visit. Given she spent less than 100 dollars over the two trips, how much (in dollars) did she spend on her first trip to the market?

p19. Consider a parallelogram $ABCD$. Let k be the line passing through A and parallel to the bisector of $\angle ABC$, and let ℓ be the bisector of $\angle BAD$. Let k intersect line CD at E and ℓ intersect line CD at F. If $AB = 13$ and $BC = 37$, find the length EF.

p20. Given for some real a, b, c, d,

$$P(x) = ax^4 + bx^3 + cx^2 + dx$$

$$P(-5) = P(-2) = P(2) = P(5) = 1$$

Find $P(10)$.

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