**Berkley mini Math Tournament Spring 2021**
www.artofproblemsolving.com/community/c2785266
by parmenides51

**Team Round** **p1.** What is the area of a triangle with side lengths $6$, $8$, and $10$?

**p2.** Let $f(n) = \sqrt{n}$. If $f(f(f(n))) = 2$, compute $n$.

**p3.** Anton is buying AguaFina water bottles. Each bottle costs $14$dollars, and Anton buys at least one water bottle. The number of dollars that Anton spends on AguaFina water bottles is a multiple of $10$. What is the least number of water bottles he can buy?

**p4.** Alex flips $3$ fair coins in a row. The probability that the first and last flips are the same can be expressed in the form $m/n$ for relatively prime positive integers $m$ and $n$. Compute $m + n$.

**p5.** How many prime numbers $p$ satisfy the property that $p^2 - 1$ is not a multiple of $6$?

**p6.** In right triangle $\vartriangle ABC$ with $AB = 5$, $BC = 12$, and $CA = 13$, point $D$ lies on $\overline{CA}$ such that $AD = BD$. The length of $CD$ can then be expressed in the form $m/n$ for relatively prime positive integers $m$ and $n$. Compute $m + n$.

**p7.** Vivienne is deciding on what courses to take for Spring $2021$, and she must choose from four math courses, three computer science courses, and five English courses. Vivienne decides that she will take one English course and two additional courses that are either computer science or math. How many choices does Vivienne have?

**p8.** Square $ABCD$ has side length $2$. Square $ACEF$ is drawn such that $B$ lies inside square $ACEF$. Compute the area of pentagon $AFECD$.

**p9.** At the Boba Math Tournament, the Blackberry Milk Team has answered $4$ out of the first $10$ questions on the Boba Round correctly. If they answer all $p$ remaining questions correctly, they will have answered exactly $\frac{9p}{5}\%$ of the questions correctly in total. How many questions are on the Boba Round?

**p10.** The sum of two positive integers is $2021$ less than their product. If one of them is a perfect square, compute the sum of the two numbers.

**p11.** Points $E$ and $F$ lie on edges $\overline{BC}$ and $\overline{DA}$ of unit square $ABCD$, respectively, such that $BE = \frac{1}{3}$ and $DF = \frac{1}{3}$ . Line segments $\overline{AE}$ and $\overline{BF}$ intersect at point $G$. The area of triangle $EFG$ can be written in the form $m/n$ , where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

**p12.** Compute the number of positive integers $n \le 2020$ for which $n^{k+1}$ is a factor of $(1 + 2 + 3 + \ + n)^k$ for some positive integer $k$.

**p13.** How many permutations of $123456$ are divisible by their last digit? For instance, $123456$ is divisible by $6$, but $561234$ is not divisible by $4$.

**p14.** Compute the sum of all possible integer values for $n$ such that $n^2 - 2n - 120$ is a positive prime number.

**p15.** Triangle $\vartriangle ABC$ has $AB = \sqrt{10}$, $BC = \sqrt{17}$, and $CA = \sqrt{41}$. The area of $\vartriangle ABC$ can be expressed in the form $m/n$ for relatively prime positive integers $m$ and $n$. Compute $m + n$.

**p16.** Let $f(x) = \frac{1+x^3+x^{10}}{1+x^{10}}$ . Compute $f(-20) + f(-19) + f(-18) + ... + f(20)$.

**p17.** Leanne and Jing Jing are walking around the $xy$-plane. In one step, Leanne can move from any point $(x, y)$ to $(x + 1, y)$ or $(x, y + 1)$ and Jing Jing can move from $(x, y)$ to $(x - 2, y + 5)$ or $(x + 3, y - 1)$. The number of ways that Leanne can move from $(0, 0)$ to $(20, 20)$ is equal to the number of ways that Jing Jing can move from $(0, 0)$ to $(a, b)$, where a and b are positive integers. Compute the minimum possible value of $a + b$.

**p18.** Compute the number positive integers $1 < k < 2021$ such that the equation $x + \sqrt{kx} = kx + \sqrt{x}$ has a positive rational solution for $x$.

**p19.** In triangle $\vartriangle ABC$, point $D$ lies on $\overline{BC}$ with $\overline{AD} \perp \overline{BC}$. If $BD = 3AD$, and the area of $\vartriangle ABC$ is $15$, then the minimum value of $AC^2$ is of the form $p\sqrt{q} - r$, where $p, q$, and $r$ are positive integers and $q$ is not divisible by the square of any prime number. Compute $p + q + r$.

**p20.** Suppose the decimal representation of $\frac{1}{n}$ is in the form $0.p_1p_2...p_j\overline{d_1d_2...d_k}$, where $p_1, ..., p_j$ , $d_1, ..., d_k$ are decimal digits, and $j$ and $k$ are the smallest possible nonnegative integers (i.e. it's possible for $j = 0$ or $k = 0$). We define the *preperiod* of $\frac{1}{n}$ to be $j$ and the *period* of $\frac{1}{n}$ to be $k$. For example, $\frac{1}{6} = 0.16666...$ has preperiod $1$ and period $1$, $\frac{1}{7} = 0.\overline{142857}$ has preperiod $0$ and period

$6$, and $\frac{1}{4} = 0.25$ has preperiod $2$ and period $0$. What is the smallest positive integer $n$ such that the sum of the preperiod and period of $\frac{1}{n}$ is $8$?

PS. You had better use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

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**Ind. Round** **p1.** What is the largest number of five dollar footlongs Jimmy can buy with 88 dollars?

**p2.** Austin, Derwin, and Sylvia are deciding on roles for BMT $2021$. There must be a single Tournament Director and a single Head Problem Writer, but one person cannot take on both roles. In how many ways can the roles be assigned to Austin, Derwin, and Sylvia?

**p3.** Sofia has$7$ unique shirts. How many ways can she place $2$ shirts into a suitcase, where the order in which Sofia places the shirts into the suitcase does not matter?

**p4.** Compute the sum of the prime factors of $2021$.

**p5.** A sphere has volume $36\pi$ cubic feet. If its radius increases by $100\%$, then its volume increases by $a\pi$ cubic feet. Compute $a$.

**p6.** The full price of a movie ticket is $\$10$, but a matinee ticket to the same movie costs only $70\%$ of the full price. If $30\%$ of the tickets sold for the movie are matinee tickets, and the total revenue from movie tickets is $\$1001$, compute the total number of tickets sold.

**p7.** Anisa rolls a fair six-sided die twice. The probability that the value Anisa rolls the second time is greater than or equal to the value Anisa rolls the first time can be expressed as $\frac{m}{n}$ , where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

**p8.** Square $ABCD$ has side length $AB = 6$. Let point $E$ be the midpoint of $\overline{BC}$. Line segments $\overline{AC}$ and $\overline{DE}$ intersect at point $F$. Compute the area of quadrilateral ABEF.

**p9.** Justine has a large bag of candy. She splits the candy equally between herself and her $4$ friends, but she needs to discard three candies before dividing so that everyone gets an equal number of candies. Justine then splits her share of the candy between herself and her two siblings, but she needs to discard one candy before dividing so that she and her siblings get an equal number of candies. If Justine had instead split all of the candy that was originally in the large bag between herself and $14$ of her classmates, what is the fewest number of candies that she would need to discard before dividing so that Justine and her $14$ classmates get an equal number of candies?

**p10.** For some positive integers $a$ and $b$, $a^2 - b^2 = 400$. If $a$ is even, compute $a$.

**p11.** Let $ABCDEFGHIJKL$ be the equilateral dodecagon shown below, and each angle is either $90^o$ or $270^o$. Let $M$ be the midpoint of $\overline{CD}$, and suppose $\overline{HM}$ splits the dodecagon into two regions. The ratio of the area of the larger region to the area of the smaller region can be expressed as $\frac{m}{n}$ , where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.
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**p12.** Nelson, who never studies for tests, takes several tests in his math class. Each test has a passing score of $60/100$. Since Nelson's test average is at least $60/100$, he manages to pass the class. If only nonnegative integer scores are attainable on each test, and Nelson gets a di erent score on every test, compute the largest possible ratio of tests failed to tests passed. Assume that for each test, Nelson either passes it or fails it, and the maximum possible score for each test is 100.

**p13.** For each positive integer $n$, let $f(n) = \frac{n}{n+1} + \frac{n+1}{n}$ . Then $f(1) + f(2) + f(3) + ... + f(10)$ can be expressed as $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

**p14.** Triangle $\triangle ABC$ has point $D$ lying on line segment $\overline{BC}$ between $B$ and $C$ such that triangle $\triangle ABD$ is equilateral. If the area of triangle $\triangle ADC$ is $\frac{1}{4}$ the area of triangle $\triangle ABC$, then $\left(\frac{AC}{AB}\right)^2$ can be expressed as $\frac{m}{n}$ , where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

**p15.** In hexagon $ABCDEF$, $AB = 60$, $AF = 40$, $EF = 20$, $DE = 20$, and each pair of adjacent edges are perpendicular to each other, as shown in the below diagram. The probability that a random point inside hexagon $ABCDEF$ is at least $20\sqrt{2}$ units away from point $D$ can be expressed in the form $\frac{a-b\pi}{c}$ , where $a, b, c$ are positive integers such that gcd$(a, b, c) = 1$. Compute $a + b + c$.
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**p16.** The equation $\sqrt{x} + \sqrt{20 - x} = \sqrt{20 + 20x - x^2}$ has $4$ distinct real solutions, $x_1$, $x_2$, $x_3$, and $x_4$. Compute $x_1 + x_2 + x_3 + x_4$.

**p17.** How many distinct words with letters chosen from $B, M, T$ have exactly $12$ distinct permutations, given that the words can be of any length, and not all the letters need to be used? For example, the word $BMMT$ has $12$ permutations. Two words are still distinct even if one is a permutation of the other. For example, $BMMT$ is distinct from $TMMB$.

**p18.** We call a positive integer binary-okay if at least half of the digits in its binary (base $2$) representation are $1$'s, but no two $1$s are consecutive. For example, $10_{10} = 1010_2$ and $5_{10} = 101_2$ are both binary-okay, but $16_{10} = 10000_2$ and $11_{10} = 1011_2$ are not. Compute the number of binary-okay positive integers less than or equal to $2020$ (in base $10$).

**p19.** A regular octahedron (a polyhedron with $8$ equilateral triangles) has side length $2$. An ant starts on the center of one face, and walks on the surface of the octahedron to the center of the opposite face in as short a path as possible. The square of the distance the ant travels can be expressed as $\frac{m}{n}$ , where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.
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**p20.** The sum of $\frac{1}{a}$ over all positive factors $a$ of the number $360$ can be expressed in the form $\frac{m}{n}$ ,where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

PS. You had better use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

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**Ind. Tie p1.** Isosceles trapezoid $ABCD$ has $AB = 2$, $BC = DA = \sqrt{17}$, and $CD = 4$. Point $E$ lies on $\overline{CD}$ such that $\overline{AE}$ splits $ABCD$ into two polygons of equal area. What is $DE$?

**p2.** At the Berkeley Sandwich Parlor, the famous BMT sandwich consists of up to five ingredients between the bread slices. These ingredients can be either bacon, mayo, or tomato, and ingredients of the same type are indistiguishable. If there must be at least one of each ingredient in the sandwich, and the order in which the ingredients are placed in the sandwich matters, how many possible ways are there to prepare a BMT sandwich?

**p3.** Three mutually externally tangent circles have radii $2$, $3$, and $3$. A fourth circle, distinct from the other three circles, is tangent to all three other circles. The sum of all possible radii of the fourth circle can be expressed as $\frac{m}{n}$ , where $m$ and $n$ are relatively prime positive integers. Compute $m + n$.

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**Pacer Round p1.** $17.5\%$ of what number is $4.5\%$ of $28000$?

**p2.** Let $x$ and $y$ be two randomly selected real numbers between $-4$ and $4$. The probability that $(x-1)(y-1)$ is positive can be written in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Compute $m+n$.

**p3.** In the $xy$-plane, Mallen is at $(-12, 7)$ and Anthony is at $(3, -14)$. Mallen runs in a straight line towards Anthony, and stops when she has traveled $\frac{2}{3}$ of the distance to Anthony. What is the sum of the $x$ and $y$ coordinates of the point that Mallen stops at?

**p4.** What are the last two digits of the sum of the first $2021$ positive integers?

**p5.** A bag has $19$ blue and $11$ red balls. Druv draws balls from the bag one at a time, without replacement. The probability that the $8$th ball he draws is red can be written in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Compute $m+n$.

**p6.** How many terms are in the arithmetic sequence $3$, $11$, ..., $779$?

**p7.** Ochama has $21$ socks and $4$ drawers. She puts all of the socks into drawers randomly, making sure there is at least $1$ sock in each drawer. If $x$ is the maximum number of socks in a single drawer, what is the difference between the maximum and minimum possible values of $x$?

**p8.** What is the least positive integer $n$ such that $\sqrt{n+1}-\sqrt{n} < \frac{1}{20}$?

**p9.** Triangle $\triangle\ ABC$ is an obtuse triangle such that $\angle ABC > 90^o$, $AB = 10$, $BC = 9$, and the area of $\triangle\ ABC$ is $36$. Compute the length of $AC$.
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**p10.** If $x+y-xy=4$, and $x$ and $y$ are integers, compute the sum of all possible values of$x+y$.

**p11.** What is the largest number of circles of radius $1$ that can be drawn inside a circle of radius $2$ such that no two circles of radius $1$ overlap?

**p12.** $22.5\%$ of a positive integer $N$ is a positive integer ending in $7$. Compute the smallest possible value of $N$.

**p13.** Alice and Bob are comparing their ages. Alice recognizes that in five years, Bob's age will be twice her age. She chuckles, recalling that five years ago, Bob's age was four times her age. How old will Alice be in five years?

**p14.** Say there is $1$ rabbit on day $1$. After each day, the rabbit population doubles, and then a rabbit dies. How many rabbits are there on day $5$?

**15.** Ajit draws a picture of a regular $63$-sided polygon, a regular $91$-sided polygon, and a regular $105$-sided polygon. What is the maximum number of lines of symmetry Ajit's picture can have?

**p16.** Grace, a problem-writer, writes $9$ out of $15$ questions on a test. A tester randomly selects $3$ of the $15$ questions, without replacement, to solve. The probability that all $3$ of the questions were written by Grace can be written in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Compute $m + n$.

**p17.** Compute the number of anagrams of the letters in $BMMTBMMT$ with no two $M$'s adjacent.

**p18.** From a $15$ inch by $15$ inch square piece of paper, Ava cuts out a heart such that the heart is a square with two semicircles attached, and the arcs of the semicircles are tangent to the edges of the piece of paper, as shown in the below diagram. The area (in square inches) of the remaining pieces of paper, after the heart is cut out and removed, can be written in the form $a - b\pi$, where $a$ and $b$ are positive integers. Compute $a + b$.

**p19.** Bayus has $2021$ marbles in a bag. He wants to place them one by one into $9$ different buckets numbered $1$ through $9$. He starts by putting the first marble in bucket $1$, the second marble in bucket $2$, the third marble in bucket $3$, etc. After placing a marble in bucket $9$, he starts back from bucket $1$ again and repeats the process. In which bucket will Bayus place the last marble in the bag?
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**p20.** What is the remainder when $1^5 + 2^5 + 3^5 + ... + 2021^5$ is divided by $5$?

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