Mediterranean Mathematics Olympiad 2016

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by cjquines0

1 Let ABC be a triangle. Let D be the intersection point of the angle bisector at A with BC.
Let T be the intersection point of the tangent line to the circumcircle of triangle ABC at point A with the line through B and C.
Let I be the intersection point of the orthogonal line to AT through point D with the altitude h_a of the triangle at point A.
Let P be the midpoint of AB, and let O be the circumcenter of triangle ABC.
Let M be the intersection point of AB and TI, and let F be the intersection point of PT and AD.
Prove: MF and AO are orthogonal to each other.

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2 Let a, b, c be positive real numbers with $a + b + c = 3$. Prove that

$$\sqrt{\frac{b}{a^2+3}} + \sqrt{\frac{c}{b^2+3}} + \sqrt{\frac{a}{c^2+3}} \le \frac{3}{2}\sqrt[4]{\frac{1}{abc}}$$

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3 Consider a 25×25 chessboard with cells $C(i, j)$ for $1 \le i, j \le 25$. Find the smallest possible number n of colors with which these cells can be colored subject to the following condition: For $1 \le i < j \le 25$ and for $1 \le s < t \le 25$, the three cells $C(i, s)$, $C(j, s)$, $C(j, t)$ carry at least two different colors.

(Proposed by Gerhard Woeginger, Austria)

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4 Determine all integers $n \ge 1$ for which the number $n^8 + n^6 + n^4 + 4$ is prime.

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