All JMBO Geometry Problems

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by Iris Aliaj, pilot, Valentin Vornicu, pbornsztein, shobber, pohoatza, Ahiles, delegat, Eukleidis, emregirgin35, Igor, Itama, neverlose, sqing, iDra36, Lukaluce

1997 Let ABC be a triangle and let I be the incenter. Let N, M be the midpoints of the sides AB and CA respectively. The lines BI and CI meet MN at K and L respectively. Prove that $AI+BI+CI>BC+KL$.

Greece

1998 Let $ABCDE$ be a convex pentagon such that $AB=AE=CD=1$, $\angle ABC=\angle DEA=90^\circ$ and $BC+DE=1$. Compute the area of the pentagon.

Greece

1999 Let ABC be a triangle with $AB=AC$. Also, let $D\in[BC]$ be a point such that $BC>BD>DC>0$, and let $\mathcal{C}_1,\mathcal{C}_2$ be the circumcircles of the triangles ABD and ADC respectively. Let BB' and CC' be diameters in the two circles, and let M be the midpoint of $B'C'$. Prove that the area of the triangle MBC is constant (i.e. it does not depend on the choice of the point D).

Greece

2000 A half-circle of diameter EF is placed on the side BC of a triangle ABC and it is tangent to the sides AB and AC in the points Q and P respectively. Prove that the intersection point K between the lines EP and FQ lies on the altitude from A of the triangle ABC.

Albania

2001 Let ABC be a triangle with $\angle C=90^\circ$ and $CA\neq CB$. Let CH be an altitude and CL be an interior angle bisector. Show that for $X\neq C$ on the line CL, we have $\angle XAC\neq\angle XBC$. Also show that for $Y\neq C$ on the line CH we have $\angle YAC\neq\angle YBC$.

Bulgaria

2001 Let ABC be an equilateral triangle and D, E points on the sides $[AB]$ and $[AC]$ respectively. If DF, EF (with $F\in AE$, $G\in AD$) are the interior angle bisectors of the angles of the triangle ADE, prove that the sum of the areas of the triangles DEF and DEG is at most equal with the area of the triangle ABC. When does the equality hold?

Greece

2002 The triangle ABC has $CA = CB$. P is a point on the circumcircle between A and B (and on the opposite side of the line AB to C). D is the foot of the perpendicular from C to PB. Show that $PA + PB = 2 \cdot PD$.

2002 Two circles with centers O_1 and O_2 meet at two points A and B such that the centers of the circles are on opposite sides of the line AB. The lines BO_1 and BO_2 meet their respective circles again at B_1 and B_2. Let M be the midpoint of B_1B_2. Let M_1, M_2 be points on the circles of centers O_1 and O_2 respectively, such that $\angle AO_1M_1 = \angle AO_2M_2$, and B_1 lies on the minor arc AM_1 while B lies on the minor arc AM_2. Show that $\angle MM_1B = \angle MM_2B$.

Ciprus

2003 Let D, E, F be the midpoints of the arcs BC, CA, AB on the circumcircle of a triangle ABC not containing the points A, B, C, respectively. Let the line DE meets BC and CA at G and H, and let M be the midpoint of the segment GH. Let the line FD meet BC and AB at K and J, and let N be the midpoint of the segment KJ.

a) Find the angles of triangle DMN;

b) Prove that if P is the point of intersection of the lines AD and EF, then the circumcenter of triangle DMN lies on the circumcircle of triangle PMN.

2004 Let ABC be an isosceles triangle with $AC = BC$, let M be the midpoint of its side AC, and let Z be the line through C perpendicular to AB. The circle through the points B, C, and M intersects the line Z at the points C and Q. Find the radius of the circumcircle of the triangle ABC in terms of $m = CQ$.

2005 Let ABC be an acute-angled triangle inscribed in a circle k. It is given that the tangent from A to the circle meets the line BC at point P. Let M be the midpoint of the line segment AP and R be the second intersection point of the circle k with the line BM. The line PR meets again the circle k at point S different from R.

Prove that the lines AP and CS are parallel.

2006 The triangle ABC is isosceles with $AB = AC$, and $\angle BAC < 60^\circ$. The points D and E are chosen on the side AC such that, $EB = ED$, and $\angle ABD \equiv \angle CBE$. Denote by O the intersection point between the internal bisectors of the angles $\angle BDC$ and $\angle ACB$. Compute $\angle COD$.

2007 Let $ABCD$ be a convex quadrilateral with $\angle DAC = \angle BDC = 36^\circ$, $\angle CBD = 18^\circ$ and $\angle BAC = 72^\circ$. The diagonals and intersect at point P . Determine the measure of $\angle APD$.

2008 The vertices A and B of an equilateral triangle ABC lie on a circle k of radius 1, and the vertex C is in the interior of the circle k. A point D, different from B, lies on k so that $AD = AB$. The line DC intersects k for the second time at point E. Find the length of the line segment CE.

2009 Let $ABCDE$ be a convex pentagon such that $AB + CD = BC + DE$ and k a circle with center on side AE that touches the sides AB, BC, CD and DE at points P, Q, R and S (different from vertices of the pentagon) respectively. Prove that lines PS and AE are parallel.

2010 Let AL and BK be angle bisectors in the non-isosceles triangle ABC (L lies on the side BC, K lies on the side AC). The perpendicular bisector of BK intersects the line AL at point M. Point N lies on the line BK such that LN is parallel to MK. Prove that $LN = NA$.

2011 Let $ABCD$ be a convex quadrilateral and points E and F on sides AB, CD such that

$$\frac{AB}{AE} = \frac{CD}{DF} = n$$

If S is the area of $AEFD$ show that $S \leq \frac{AB \cdot CD + n(n-1)AD^2 + n^2 DA \cdot BC}{2n^2}$

2012 Let the circles k_1 and k_2 intersect at two points A and B, and let t be a common tangent of k_1 and k_2 that touches k_1 and k_2 at M and N respectively. If $t \perp AM$ and $MN = 2AM$, evaluate the angle NMB.

2013 Let ABC be an acute-angled triangle with $AB < AC$ and let O be the centre of its circumcircle ω. Let D be a point on the line segment BC such that $\angle BAD = \angle CAO$. Let E be the second point of intersection of ω and the line AD. If M, N and P are the midpoints of the line segments BE, OD and AC, respectively, show that the points M, N and P are collinear.

2014 Consider an acute triangle ABC of area S. Let $CD \perp AB$ ($D \in AB$), $DM \perp AC$ ($M \in AC$) and $DN \perp BC$ ($N \in BC$). Denote by H_1 and H_2 the orthocentres of the triangles MNC, respectively MND. Find the area of the quadrilateral AH_1BH_2 in terms of S.

2015 Let ABC be an acute triangle.The lines l_1 and l_2 are perpendicular to AB at the points A and B, respectively.The perpendicular lines from the midpoint M of AB to the lines AC and BC intersect l_1 and l_2 at the points E and F, respectively.If D is the intersection point of the lines EF and MC, prove that

$$\angle ADB = \angle EMF.$$

2016 A trapezoid $ABCD$ ($AB||CF$,$AB > CD$) is circumscribed.The incircle of the triangle ABC touches the lines AB and AC at the points M and N,respectively.Prove that the incenter of the trapezoid $ABCD$ lies on the line MN.

2017 Let ABC be an acute triangle such that $AB \neq AC$,with circumcircle Γ and circumcenter O. Let M be the midpoint of BC and D be a point on Γ such that $AD \perp BC$. let T be a point such that $BDCT$ is a parallelogram and Q a point on the same side of BC as A such that $\angle BQM = \angle BCA$ and $\angle CQM = \angle CBA$. Let the line AO intersect Γ at E $(E \neq A)$ and let the circumcircle of $\triangle ETQ$ intersect Γ at point $X \neq E$. Prove that the point A, M and X are collinear.

2018 Let $\triangle ABC$ and A',B',C' the symmetrics of vertex over opposite sides.The intersection of the circumcircles of $\triangle ABB'$ and $\triangle ACC'$ is A_1.B_1 and C_1 are defined similarly.Prove that lines AA_1,BB_1 and CC_1 are concurent.

2019 Triangle ABC is such that $AB < AC$. The perpendicular bisector of side BC intersects lines AB and AC at points P and Q, respectively. Let H be the orthocentre of triangle ABC, and let M and N be the midpoints of segments BC and PQ, respectively. Prove that lines HM and AN meet on the circumcircle of ABC.

2020 Let $\triangle ABC$ be a right-angled triangle with $\angle BAC = 90^\circ$ and let E be the foot of the perpendicular from A to BC. Let $Z \neq A$ be a point on the line AB with $AB = BZ$. Let (c) be the circumcircle of the triangle $\triangle AEZ$. Let D be the second point of intersection of (c) with ZC and let F be the antidiametric point of D with respect to (c). Let P be the point of intersection of the lines FE and CZ. If the tangent to (c) at Z meets PA at T, prove that the points T, E, B, Z are concyclic.

Proposed by *Theoklitos Parayiou, Cyprus*

2021 Let ABC be an acute scalene triangle with circumcenter O. Let D be the foot of the altitude from A to the side BC. The lines BC and AO intersect at E. Let s be the line through E perpendicular to AO. The line s intersects AB and AC at K and L, respectively. Denote by ω the circumcircle of triangle AKL. Line AD intersects ω again at X.
Prove that ω and the circumcircles of triangles ABC and DEX have a common point.
