Exeter Math Club Competition - Team Round, years 2010-23

www.artofproblemsolving.com/community/c2825541

by parmenides51

2010 **p1.** A very large lucky number N consists of eighty-eight 8s in a row. Find the remainder when this number N is divided by 6.

p2. If 3 chickens can lay 9 eggs in 4 days, how many chickens does it take to lay 180 eggs in 8 days?

p3. Find the ordered pair (x, y) of real numbers satisfying the conditions $x > y$, $x + y = 10$, and $xy = -119$.

p4. There is pair of similar triangles. One triangle has side lengths $4, 6$, and 9. The other triangle has side lengths 8, 12 and x. Find the sum of two possible values of x.

p5. If $x^2 + \frac{1}{x^2} = 3$, there are two possible values of $x + \frac{1}{x}$. What is the smaller of the two values?

p6. Three flavors (chocolate strawberry, vanilla) of ice cream are sold at Brian's ice cream shop. Brian's friend Zerg gets a coupon for 10 free scoops of ice cream. If the coupon requires Zerg to choose an even number of scoops of each flavor of ice cream, how many ways can he choose his ice cream scoops? (For example, he could have 6 scoops of vanilla and 4 scoops of chocolate. The order in which Zerg eats the scoops does not matter.)

p7. David decides he wants to join the West African Drumming Ensemble, and thus he goes to the store and buys three large cylindrical drums. In order to ensure none of the drums drop on the way home, he ties a rope around all of the drums at their mid sections so that each drum is next to the other two. Suppose that each drum has a diameter of 3.5 feet. David needs m feet of rope. Given that $m = a\pi + b$, where a and b are rational numbers, find sum $a + b$.

p8. Segment AB is the diameter of a semicircle of radius 24. A beam of light is shot from a point $12\sqrt{3}$ from the center of the semicircle, and perpendicular to AB. How many times does it reflect off the semicircle before hitting AB again?

p9. A cube is inscribed in a sphere of radius 8. A smaller sphere is inscribed in the same sphere such that it is externally tangent to one face of the cube and internally tangent to the larger sphere. The maximum value of the ratio of the volume of the smaller sphere to the volume of the larger sphere can be written in the form $\frac{a-\sqrt{b}}{36}$, where a and b are positive integers. Find the product ab.

p10. How many ordered pairs (x, y) of integers are there such that $2xy + x + y = 52$?

p11. Three musketeers looted a caravan and walked off with a chest full of coins. During the night, the first musketeer divided the coins into three equal piles, with one coin left over. He threw it into the ocean and took one of the piles for himself, then went back to sleep. The second musketeer woke up an hour later. He divided the remaining coins into three equal piles, and threw out the one coin that was left over. He took one of the piles and went back to sleep. The third musketeer woke up and divided the remaining coins into three equal piles, threw out the extra coin, and took one pile for himself. The next morning, the three musketeers gathered around to divide the coins into three equal piles. Strangely enough, they had one coin left over this time as well. What is the minimum number of coins that were originally in the chest?

p12. The diagram shows a rectangle that has been divided into ten squares of different sizes. The smallest square is 2×2 (marked with *). What is the area of the rectangle (which looks rather like a square itself)?
`https://cdn.artofproblemsolving.com/attachments/4/a/7b8ebc1a9e3808096539154f0107f3e23d168`
`png`

p13. Let $A = (3, 2)$, $B = (0, 1)$, and P be on the line $x + y = 0$. What is the minimum possible value of $AP + BP$?

p14. Mr. Mustafa the number man got a $6 \times x$ rectangular chess board for his birthday. Because he was bored, he wrote the numbers 1 to $6x$ starting in the upper left corner and moving across row by row (so the number $x+1$ is in the 2nd row, 1st column). Then, he wrote the same numbers starting in the upper left corner and moving down each column (so the number 7 appears in the 1st row, 2nd column). He then added up the two numbers in each of the cells and found that some of the sums were repeated. Given that x is less than or equal to 100, how many possibilities are there for x?

p15. Six congruent equilateral triangles are arranged in the plane so that every triangle shares at least one whole edge with some other triangle. Find the number of distinct arrangements. (Two arrangements are considered the same if one can be rotated and/or reflected onto another.)

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2011

p1. Velociraptor A is located at $x = 10$ on the number line and runs at 4 units per second. Velociraptor B is located at $x = -10$ on the number line and runs at 3 units per second. If the velociraptors run towards each other, at what point do they meet?

p2. Let n be a positive integer. There are n non-overlapping circles in a plane with radii $1, 2, ..., n$. The total area that they enclose is at least 100. Find the minimum possible value of n.

p3. How many integers between 1 and 50, inclusive, are divisible by 4 but not 6?

p4. Let $a \star b = 1 + \frac{b}{a}$. Evaluate $((((((1 \star 1) \star 1) \star 1) \star 1) \star 1) \star 1) \star 1$.

p5. In acute triangle ABC, D and E are points inside triangle ABC such that $DE \parallel BC$, B is closer to D than it is to E, $\angle AED = 80^o$, $\angle ABD = 10^o$, and $\angle CBD = 40^o$. Find the measure of $\angle BAE$, in degrees.

p6. Al is at $(0, 0)$. He wants to get to $(4, 4)$, but there is a building in the shape of a square with vertices at $(1, 1)$, $(1, 2)$, $(2, 2)$, and $(2, 1)$. Al cannot walk inside the building. If Al is not restricted to staying on grid lines, what is the shortest distance he can walk to get to his destination?

p7. Point $A = (1, 211)$ and point $B = (b, 2011)$ for some integer b. For how many values of b is the slope of AB an integer?

p8. A palindrome is a number that reads the same forwards and backwards. For example, 1, 11 and 141 are all palindromes. How many palindromes between 1 and 1000 are divisible by 11?

p9. Suppose x, y, z are real numbers that satisfy:

$$x + y - z = 5$$

$$y + z - x = 7$$

$$z + x - y = 9$$

Find $x^2 + y^2 + z^2$.

p10. In triangle ABC, $AB = 3$ and $AC = 4$. The bisector of angle A meets BC at D. The line through D perpendicular to AD intersects lines AB and AC at F and E, respectively. Compute $EC - FB$. (See the following diagram.)
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png

p11. Bob has a six-sided die with a number written on each face such that the sums of the numbers written on each pair of opposite faces are equal to each other. Suppose that the numbers 109, 131, and 135 are written on three faces which share a corner. Determine the maximum possible sum of the numbers on the three remaining faces, given that all three are positive primes less than 200.

p12. Let d be a number chosen at random from the set $\{142, 143, ..., 198\}$. What is the probability that the area of a rectangle with perimeter 400 and diagonal length d is an integer?

p13. There are 3 congruent circles such that each circle passes through the centers of the other two. Suppose that A, B, and C are points on the circles such that each circle has exactly one of A, B, or C on it and triangle ABC is equilateral. Find the ratio of the maximum possible area of ABC to the minimum possible area of ABC. (See the following diagram.)
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png

p14. Let k and m be constants such that for all triples (a, b, c) of positive real numbers,

$$\sqrt{\frac{4}{a^2}+\frac{36}{b^2}+\frac{9}{c^2}+\frac{k}{ab}}=\left|\frac{2}{a}+\frac{6}{b}+\frac{3}{c}\right|$$

if and only if $am^2 + bm + c = 0$. Find k.

p15. A bored student named Abraham is writing n numbers $a_1, a_2, ..., a_n$. The value of each number is either $1, 2$, or 3; that is, a_i is $1, 2$ or 3 for $1 \le i \le n$. Abraham notices that the ordered triples

$$(a_1, a_2, a_3), (a_2, a_3, a_4), ..., (a_{n-2}, a_{n-1}, a_n), (a_{n-1}, a_n, a_1), (a_n, a_1, a_2)$$

are distinct from each other. What is the maximum possible value of n? Give the answer n, along with an example of such a sequence. Write your answer as an ordered pair. (For example, if the answer were 5, you might write $(5, 12311)$.)

PS. You had better use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2012 **p1.** The longest diagonal of a regular hexagon is 12 inches long. What is the area of the hexagon, in square inches?

p2. When Al and Bob play a game, either Al wins, Bob wins, or they tie. The probability that Al does not win is $\frac{2}{3}$, and the probability that Bob does not win is $\frac{3}{4}$. What is the probability that they tie?

p3. Find the sum of the $a + b$ values over all pairs of integers (a, b) such that $1 \le a < b \le 10$. That is, compute the sum

$$(1 + 2) + (1 + 3) + (1 + 4) + ... + (2 + 3) + (2 + 4) + ... + (9 + 10).$$

p4. A 3×11 cm rectangular box has one vertex at the origin, and the other vertices are above the x-axis. Its sides lie on the lines $y = x$ and $y = -x$. What is the y-coordinate of the highest point on the box, in centimeters?

p5. Six blocks are stacked on top of each other to create a pyramid, as shown below. Carl removes blocks one at a time from the pyramid, until all the blocks have been removed. He never removes a block until all the blocks that rest on top of it have been removed. In how many different orders can Carl remove the blocks?
`https://cdn.artofproblemsolving.com/attachments/b/e/9694d92eeb70b4066b1717fedfbfc601631ce`
`png`

p6. Suppose that a right triangle has sides of lengths $\sqrt{a+b\sqrt3}$,$\sqrt{3+2\sqrt3}$, and $\sqrt{4+5\sqrt3}$, where a, b are positive integers. Find all possible ordered pairs (a, b).

p7. Farmer Chong Gu glues together 4 equilateral triangles of side length 1 such that their edges coincide. He then drives in a stake at each vertex of the original triangles and puts a rubber band around all the stakes. Find the minimum possible length of the rubber band.

p8. Compute the number of ordered pairs (a, b) of positive integers less than or equal to 100, such that a $b - 1$ is a multiple of 4.

p9. In triangle ABC, $\angle C = 90^o$. Point P lies on segment BC and is not B or C. Point I lies on segment AP. If $\angle BIP = \angle PBI = \angle CAB = m^o$ for some positive integer m, find the sum of all possible values of m.

p10. Bob has 2 identical red coins and 2 identical blue coins, as well as 4 distinguishable buckets. He places some, but not necessarily all, of the coins into the buckets such that no bucket contains two coins of the same color, and at least one bucket is not empty. In how many ways can he do this?

p11. Albert takes a 4×4 checkerboard and paints all the squares white. Afterward, he wants to paint some of the square black, such that each square shares an edge with an odd number of black squares. Help him out by drawing one possible configuration in which this holds. (Note: the answer sheet contains a 4×4 grid.)

p12. Let S be the set of points (x, y) with $0 \le x \le 5$, $0 \le y \le 5$ where x and y are integers. Let T be the set of all points in the plane that are the midpoints of two distinct points in S. Let U be the set of all points in the plane that are the midpoints of two distinct points in T. How many distinct points are in U? (Note: The points in T and U do not necessarily have integer coordinates.)

p13. In how many ways can one express 6036 as the sum of at least two (not necessarily positive) consecutive integers?

p14. Let a, b, c, d, e, f be integers (not necessarily distinct) between -100 and 100, inclusive, such that $a + b + c + d + e + f = 100$. Let M and m be the maximum and minimum possible values, respectively, of
$$abc + bcd + cde + def + efa + fab + ace + bdf.$$
Find $\frac{M}{m}$.

p15. In quadrilateral $ABCD$, diagonal AC bisects diagonal BD. Given that $AB = 20$, $BC = 15$, $CD = 13$, $AC = 25$, find DA.

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2013 **p1.** Determine the number of ways to place 4 rooks on a 4×4 chessboard such that:
(a) no two rooks attack one another, and
(b) the main diagonal (the set of squares marked X below) does not contain any rooks.
`https://cdn.artofproblemsolving.com/attachments/e/e/e3aa96de6c8ed468c6ef3837e66a0bce360d3`
`png`
The rooks are indistinguishable and the board cannot be rotated. (Two rooks attack each other if they are in the same row or column.)

p2. Seven students, numbered 1 to 7 in counter-clockwise order, are seated in a circle. Fresh Mann has 100 erasers, and he wants to distribute them to the students, albeit unfairly. Starting with person 1 and proceeding counter-clockwise, Fresh Mann gives i erasers to student i; for example, he gives 1 eraser to student 1, then 2 erasers to student 2, et cetera. He continues around the circle until he does not have enough erasers to give to the next person. At this point, determine the number of erasers that Fresh Mann has.

p3. Let ABC be a triangle with $AB = AC = 17$ and $BC = 24$. Approximate $\angle ABC$ to the nearest multiple of 10 degrees.

p4. Define a sequence of rational numbers $\{x_n\}$ by $x_1 = \frac{3}{5}$ and for $n \ge 1$, $x_{n+1} = 2 - \frac{1}{x_n}$. Compute the product $x_1x_2x_3...x_{2013}$.

p5. In equilateral triangle ABC, points P and R lie on segment AB, points I and M lie on segment BC, and points E and S lie on segment CA such that $PRIMES$ is a equiangular hexagon. Given that $AB = 11$, $PR = 2$, $IM = 3$, and $ES = 5$, compute the area of hexagon $PRIMES$.

p6. Let $f(a, b) = \frac{a^2}{a+b}$. Let A denote the sum of $f(i, j)$ over all pairs of integers (i, j) with $1 \le i < j \le 10$; that is,

$$A = (f(1, 2) + f(1, 3) + ... + f(1, 10)) + (f(2, 3) + f(2, 4) + ... + f(2, 10)) + ... + f(9, 10).$$

Similarly, let B denote the sum of $f(i, j)$ over all pairs of integers (i, j) with $1 \le j < i \le 10$, that is,

$$B = (f(2, 1) + f(3, 1) + ... + f(10, 1)) + (f(3, 2) + f(4, 2) + ... + f(10, 2)) + ... + f(10, 9).$$

Compute $B - A$.

p7. Fresh Mann has a pile of seven rocks with weights $1, 1, 2, 4, 8, 16$, and 32 pounds and some integer X between 1 and 64, inclusive. He would like to choose a set of the rocks whose total weight is exactly X pounds. Given that he can do so in more than one way, determine the sum of all possible values of X. (The two 1-pound rocks are indistinguishable.)

p8. Let $ABCD$ be a convex quadrilateral with $AB = BC = CA$. Suppose that point P lies inside the quadrilateral with $AP = PD = DA$ and $\angle PCD = 30^o$. Given that $CP = 2$ and $CD = 3$, compute CA.

p9. Define a sequence of rational numbers $\{x_n\}$ by $x_1 = 2$, $x_2 = \frac{13}{2}$, and for $n \ge 1$, $x_{n+2} = 3 - \frac{3}{x_{n+1}} + \frac{1}{x_nx_{n+1}}$. Compute x_{100}.

p10. Ten prisoners are standing in a line. A prison guard wants to place a hat on each prisoner. He has two colors of hats, red and blue, and he has 10 hats of each color. Determine the number of ways in which the prison guard can place hats such that among any set of consecutive prisoners, the number of prisoners with red hats and the number of prisoners with blue hats differ by at most 2.

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2014 **p1.** What is the units digit of the product of the first seven primes?

p2. In triangle ABC, $\angle BAC$ is a right angle and $\angle ACB$ measures 34 degrees. Let D be a point on segment BC for which $AC = CD$, and let the angle bisector of $\angle CBA$ intersect line AD at E. What is the measure of $\angle BED$?

p3. Chad numbers five paper cards on one side with each of the numbers from 1 through 5. The cards are then turned over and placed in a box. Jordan takes the five cards out in random order and again numbers them from 1 through 5 on the other side. When Chad returns to look at the cards, he deduces with great difficulty that the probability that exactly two of the cards have the same number on both sides is p. What is p?

p4. Only one real value of x satisfies the equation $kx^2 + (k + 5)x + 5 = 0$. What is the product of all possible values of k?

p5. On the Exeter Space Station, where there is no effective gravity, Chad has a geometric model consisting of 125 wood cubes measuring 1 centimeter on each edge arranged in a 5 by 5 by 5 cube. An aspiring carpenter, he practices his trade by drawing the projection of the model from three views: front, top, and side. Then, he removes some of the original 125 cubes and redraws the three projections of the model. He observes that his three drawings after removing some cubes are identical to the initial three. What is the maximum number of cubes that he could have removed? (Keep in mind that the cubes could be suspended without support.)

p6. Eric, Meena, and Cameron are studying the famous equation $E = mc^2$. To memorize this formula, they decide to play a game. Eric and Meena each randomly think of an integer between 1 and 50, inclusively, and substitute their numbers for E and m in the equation. Then, Cameron solves for the absolute value of c. What is the probability that Cameron's result is a rational number?

p7. Let CDE be a triangle with side lengths $EC = 3$, $CD = 4$, and $DE = 5$. Suppose that points A and B are on the perimeter of the triangle such that line AB divides the triangle into two polygons of equal area and perimeter. What are all the possible values of the length of segment AB?

p8. Chad and Jordan are raising bacteria as pets. They start out with one bacterium in a Petri dish. Every minute, each existing bacterium turns into $0, 1, 2$ or 3 bacteria, with equal probability for each of the four outcomes. What is the probability that the colony of bacteria will eventually die out?

p9. Let $a = w+x$, $b = w+y$, $c = x+y$, $d = w+z$, $e = x+z$, and $f = y+z$. Given that $af = be = cd$ and

$$(x-y)(x-z)(x-w)+(y-x)(y-z)(y-w)+(z-x)(z-y)(z-w)+(w-x)(w-y)(w-z) = 1,$$

what is

$$2(a^2+b^2+c^2+d^2+e^2+f^2)-ab-ac-ad-ae-bc-bd-bf-ce-cf-de-df-ef?$$

p10. If a and b are integers at least 2 for which a^b-1 strictly divides b^a-1, what is the minimum possible value of ab?
Note: If x and y are integers, we say that x strictly divides y if x divides y and $|x| \neq |y|$.

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2015 **p1.** Nicky is studying biology and has a tank of 17 lizards. In one day, he can either remove 5 lizards or add 2 lizards to his tank. What is the minimum number of days necessary for Nicky to get rid of all of the lizards from his tank?

p2. What is the maximum number of spheres with radius 1 that can fit into a sphere with radius 2?

p3. A positive integer x is sunny if $3x$ has more digits than x. If all sunny numbers are written in increasing order, what is the 50th number written?

p4. Quadrilateral $ABCD$ satisfies $AB = 4$, $BC = 5$, $DA = 4$, $\angle DAB = 60^o$, and $\angle ABC = 150^o$. Find the area of $ABCD$.

p5. Totoro wants to cut a 3 meter long bar of mixed metals into two parts with equal monetary value. The left meter is bronze, worth 10 zoty per meter, the middle meter is silver, worth 25 zoty per meter, and the right meter is gold, worth 40 zoty per meter. How far, in meters, from the left should Totoro make the cut?

p6. If the numbers x_1, x_2, x_3, x_4, and $x5$ are a permutation of the numbers $1, 2, 3, 4$, and 5, compute the maximum possible value of

$$|x_1 - x_2| + |x_2 - x_3| + |x_3 - x_4| + |x_4 - x_5|.$$

p7. In a 3×4 grid of 12 squares, find the number of paths from the top left corner to the bottom right corner that satisfy the following two properties: • The path passes through each square exactly once. • Consecutive squares share a side.
Two paths are considered distinct if and only if the order in which the twelve squares are visited is different. For instance, in the diagram below, the two paths drawn are considered the same.
`https://cdn.artofproblemsolving.com/attachments/7/a/bb3471bbde1a8f58a61d9dd69c8527cfece05`
`png`

p8. Scott, Demi, and Alex are writing a computer program that is 25 ines long. Since they are working together on one computer, only one person may type at a time. To encourage collaboration, no person can type two lines in a row, and everyone must type something. If Scott takes 10 seconds to type one line, Demi takes 15 seconds, and Alex takes 20 seconds, at least how long, in seconds, will it take them to finish the program?

p9. A hand of four cards of the form $(c, c, c + 1, c + 1)$ is called a tractor. Vinjai has a deck consisting of four of each of the numbers 7, 8, 9 and 10. If Vinjai shuffles and draws four cards from his deck, compute the probability that they form a tractor.

p10. The parabola $y = 2x^2$ is the wall of a fortress. Totoro is located at $(0, 4)$ and fires a cannonball in a straight line at the closest point on the wall. Compute the y-coordinate of the point on the wall that the cannonball hits.

p11. How many ways are there to color the squares of a 10 by 10 grid with black and white such that in each row and each column there are exactly two black squares and between the two black squares in a given row or column there are exactly **4** white squares? Two configurations that are the same under rotations or reflections are considered different.

p12. In rectangle $ABCD$, points E and F are on sides AB and CD, respectively, such that $AE = CF > AD$ and $\angle CED = 90^o$. Lines AF, BF, CE and DE enclose a rectangle whose area is 24% of the area of $ABCD$. Compute $\frac{BF}{CE}$.

p13. Link cuts trees in order to complete a quest. He must cut 3 Fenwick trees, 3 Splay trees and 3 KD trees. If he must also cut 3 trees of the same type in a row at some point during his quest, in how many ways can he cut the trees and complete the quest? (Trees of the same type are indistinguishable.)

p14. Find all ordered pairs (a, b) of positive integers such that $\sqrt{64a+b^2}+8=8\sqrt{a}+b$.

p15. Let $ABCDE$ be a convex pentagon such that $\angle ABC=\angle BCD=108^o$, $\angle CDE=168^o$ and $AB=BC=CD=DE$. Find the measure of $\angle AEB$

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2016 **p1.** Lisa is playing the piano at a tempo of 80 beats per minute. If four beats make one measure of her rhythm, how many seconds are in one measure?

p2. Compute the smallest integer $n>1$ whose base-2 and base-3 representations both do not contain the digit 0.

p3. In a room of 24 people, $5/6$ of the people are old, and $5/8$ of the people are male. At least how many people are both old and male?

p4. Juan chooses a random even integer from 1 to 15 inclusive, and Gina chooses a random odd integer from 1 to 15 inclusive. What is the probability that Juan's number is larger than Gina's number? (They choose all possible integers with equal probability.)

p5. Set S consists of all positive integers less than or equal to 2016. Let A be the subset of S consisting of all multiples of 6. Let B be the subset of S consisting of all multiples of 7. Compute the ratio of the number of positive integers in A but not B to the number of integers in B but not A.

p6. Three peas form a unit equilateral triangle on a flat table. Sebastian moves one of the peas a distance d along the table to form a right triangle. Determine the minimum possible value of d.

p7. Oumar is four times as old as Marta. In m years, Oumar will be three times as old as Marta will be. In another n years after that, Oumar will be twice as old as Marta will be. Compute the ratio m/n.

p8. Compute the area of the smallest square in which one can inscribe two non-overlapping equilateral triangles with side length 1.

p9. Teemu, Marcus, and Sander are signing documents. If they all work together, they would finish in 6 hours. If only Teemu and Sander work together, the work would be finished in 8 hours. If only Marcus and Sander work together, the work would be finished in 10 hours. How many hours would Sander take to finish signing if he worked alone?

p10. Triangle ABC has a right angle at B. A circle centered at B with radius BA intersects side AC at a point D different from A. Given that $AD = 20$ and $DC = 16$, find the length of BA.

p11. A regular hexagon H with side length 20 is divided completely into equilateral triangles with side length 1. How many regular hexagons with sides parallel to the sides of H are formed by lines in the grid?

p12. In convex pentagon $PEARL$, quadrilateral $PERL$ is a trapezoid with side PL parallel to side ER. The areas of triangle ERA, triangle LAP, and trapezoid $PERL$ are all equal. Compute the ratio $\frac{PL}{ER}$.

p13. Let m and n be positive integers with $m < n$. The first two digits after the decimal point in the decimal representation of the fraction m/n are 74. What is the smallest possible value of n?

p14. Define functions $f(x, y) = \frac{x+y}{2} - \sqrt{xy}$ and $g(x, y) = \frac{x+y}{2} + \sqrt{xy}$. Compute $g(g(f(1, 3), f(5, 7)), g(f(3, 5), f$

p15. Natalia plants two gardens in a 5×5 grid of points. Each garden is the interior of a rectangle with vertices on grid points and sides parallel to the sides of the grid. How many unordered pairs of two non-overlapping rectangles can Nataliia choose as gardens? (The two rectangles may share an edge or part of an edge but should not share an interior point.)

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2017 **p1.** Compute $2017 + 7201 + 1720 + 172$.

p2. A number is called *downhill* if its digits are distinct and in descending order. (For example, 653 and 8762 are downhill numbers, but 97721 is not.) What is the smallest downhill number greater than 86432?

p3. Each vertex of a unit cube is sliced off by a planar cut passing through the midpoints of the three edges containing that vertex. What is the ratio of the number of edges to the number of faces of the resulting solid?

p4. In a square with side length 5, the four points that divide each side into five equal segments are marked. Including the vertices, there are 20 marked points in total on the boundary of the square. A pair of distinct points A and B are chosen randomly among the 20 points. Compute the probability that $AB = 5$.

p5. A positive two-digit integer is one less than five times the sum of its digits. Find the sum of all possible such integers.

p6. Let
$$f(x) = 5^{4^{3^{2^x}}}.$$
Determine the greatest possible value of L such that $f(x) > L$ for all real numbers x.

p7. If $\overline{AAAA}+\overline{BB} = \overline{ABCD}$ for some distinct base-10 digits A, B, C, D that are consecutive in some order, determine the value of $ABCD$. (The notation $\overline{ABCD}$ refers to the four-digit integer with thousands digit A, hundreds digit B, tens digit C, and units digit D.)

p8. A regular tetrahedron and a cube share an inscribed sphere. What is the ratio of the volume of the tetrahedron to the volume of the cube?

p9. Define $\lfloor x\rfloor$ as the greatest integer less than or equal to x, and $x = x - \lfloor x\rfloor$ as the fractional part of x. If $\lfloor x^2\rfloor = 2\lfloor x\rfloor$ and $\{x^2\} = \frac12\{x\}$, determine all possible values of x.

p10. Find the largest integer $N > 1$ such that it is impossible to divide an equilateral triangle of side length 1 into N smaller equilateral triangles (of possibly different sizes).

p11. Let f and g be two quadratic polynomials. Suppose that f has zeroes 2 and 7, g has zeroes 1 and 8, and $f - g$ has zeroes 4 and 5. What is the product of the zeroes of the polynomial $f + g$?

p12. In square $PQRS$, points A, B, C, D, E, and F are chosen on segments PQ, QR, PR, RS, SP, and PR, respectively, such that $ABCDEF$ is a regular hexagon. Find the ratio of the area of $ABCDEF$ to the area of $PQRS$.

p13. For positive integers m and n, define $f(m, n)$ to be the number of ways to distribute m identical candies to n distinct children so that the number of candies that any two children receive differ by at most 1. Find the number of positive integers n satisfying the equation $f(2017, n) = f(7102, n)$.

p14. Suppose that real numbers x and y satisfy the equation

$$x^4 + 2x^2y^2 + y^4 - 2x^2 + 32xy - 2y^2 + 49 = 0.$$

Find the maximum possible value of $\frac{y}{x}$.

p15. A point P lies inside equilateral triangle ABC. Let A', B', C' be the feet of the perpendiculars from P to BC, AC, AB, respectively. Suppose that $PA = 13$, $PB = 14$, and $PC = 15$. Find the area of $A'B'C'$.

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2018 **p1.** Farmer James goes to Kristy's Krispy Chicken to order a crispy chicken sandwich. He can choose from 3 types of buns, 2 types of sauces, 4 types of vegetables, and 4 types of cheese. He can only choose one type of bun and cheese, but can choose any nonzero number of sauces, and the same with vegetables. How many different chicken sandwiches can Farmer James order?

p2. A line with slope 2 and a line with slope 3 intersect at the point (m, n), where $m, n > 0$. These lines intersect the x axis at points A and B, and they intersect the y axis at points C and D. If $AB = CD$, find m/n.

p3. A multi-set of 11 positive integers has a median of 10, a unique mode of 11, and a mean of 12. What is the largest possible number that can be in this multi-set? (A multi-set is a set that allows repeated elements.)

p4. Farmer James is swimming in the Eggs-Eater River, which flows at a constant rate of 5 miles per hour, and is recording his time. He swims 1 mile upstream, against the current, and then swims 1 mile back to his starting point, along with the current. The time he recorded was double the time that he would have recorded if he had swum in still water the entire trip. To the nearest integer, how fast can Farmer James swim in still water, in miles per hour?

p5. $ABCD$ is a square with side length 60. Point E is on AD and F is on CD such that $\angle BEF = 90^o$. Find the minimum possible length of CF.

p6. Farmer James makes a trianglomino by gluing together 5 equilateral triangles of side length

1, with adjacent triangles sharing an entire edge. Two trianglominoes are considered the same if they can be matched using only translations and rotations (but not reflections). How many distinct trianglominoes can Farmer James make?

p7. Two real numbers x and y satisfy $x^2 - y^2 = 2y - 2x$, and $x + 6 = y^2 + 2y$. What is the sum of all possible values of$y$?

p8. Let N be a positive multiple of 840. When N is written in base 6, it is of the form $\overline{abcdef}_6$ where a, b, c, d, e, f are distinct base 6 digits. What is the smallest possible value of N, when written in base 6?

p9. For $S = \{1, 2, ..., 12\}$, find the number of functions $f : S \to S$ that satisfy the following 3 conditions:
(a) If n is divisible by 3, $f(n)$ is not divisible by 3,
(b) If n is not divisible by 3, $f(n)$ is divisible by 3, and
(c) $f(f(n)) = n$ holds for exactly 8 distinct values of n in S.

p10. Regular pentagon $JAMES$ has area 1. Let O lie on line EM and N lie on line MA so that E, M, O and M, A, N lie on their respective lines in that order. Given that $MO = AN$ and $NO = 11 \cdot ME$, find the area of NOM.

p11. Hen Hao is flipping a special coin, which lands on its sunny side and its rainy side each with probability $1/2$. Hen Hao flips her coin ten times. Given that the coin never landed with its rainy side up twice in a row, find the probability that Hen Hao's last flip had its sunny side up.

p12. Find the product of all integer values of a such that the polynomial $x^4 + 8x^3 + ax^2 + 2x - 1$ can be factored into two non-constant polynomials with integer coefficients.

p13. Isosceles trapezoid $ABCD$ has $AB = CD$ and $AD = 6BC$. Point X is the intersection of the diagonals AC and BD. There exist a positive real number k and a point P inside $ABCD$ which satisfy

$$[PBC] : [PCD] : [PDA] = 1 : k : 3,$$

where $[XYZ]$ denotes the area of triangle XYZ. If $PX \parallel AB$, find the value of k.

p14. How many positive integers $n < 1000$ are there such that in base 10, every digit in $3n$ (that isn't a leading zero) is greater than the corresponding place value digit (possibly a leading zero) in n? For example, $n = 56$, $3n = 168$ satisfies this property as $1 > 0$, $6 > 5$, and $8 > 6$. On the other hand, $n = 506$, $3n = 1518$ does not work because of the hundreds place.

p15. Find the greatest integer that is smaller than

$$\frac{2018}{37^2}+\frac{2018}{39^2}+...+\frac{2018}{107^2}.$$

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2019 **p1.** Three positive integers sum to 16. What is the least possible value of the sum of their squares?

p2. Ben is thinking of an odd positive integer less than 1000. Ben subtracts 1 from his number and divides by 2, resulting in another number. If his number is still odd, Ben repeats this procedure until he gets an even number. Given that the number he ends on is 2, how many possible values are there for Ben's original number?

p3. Triangle ABC is isosceles, with $AB = BC = 18$ and has circumcircle ω. Tangents to ω at A and B intersect at point D. If $AD = 27$, what is the length of AC?

p4. How many non-decreasing sequences of five natural numbers have first term 1, last term 11, and have no three terms equal?

p5. Adam is bored, and has written the string "EMCC" on a piece of paper. For fun, he decides to erase every letter "C", and replace it with another instance of "EMCC". For example, after one step, he will have the string "EMEMCCEMCC". How long will his string be after 8 of these steps?

p6. Eric has two coins, which land heads 40% and 60% of the time respectively. He chooses a coin randomly and flips it four times. Given that the first three flips contained two heads and one tail, what is the probability that the last flip was heads?

p7. In a five person rock-paper-scissors tournament, each player plays against every other player exactly once, with each game continuing until one player wins. After each game, the winner gets 1 point, while the loser gets no points. Given that each player has a 50% chance of defeating any other player, what is the probability that no two players end up with the same amount of points?

p8. Let $\triangle ABC$ have $\angle A = \angle B = 75^o$. Points D, E, and F are on sides BC, CA, and AB, respectively, so that EF is parallel to BC, $EF \perp DE$, and $DE = EF$. Find the ratio of $\triangle DEF$'s area to $\triangle ABC$'s area.

p9. Suppose a, b, c are positive integers such that $a+b = \sqrt{c^2 + 336}$ and $a-b = \sqrt{c^2 - 336}$. Find $a + b + c$.

p10. How many times on a 12-hour analog clock are there, such that when the minute and hour hands are swapped, the result is still a valid time? (Note that the minute and hour hands move continuously, and don't always necessarily point to exact minute/hour marks.)

p11. Adam owns a square S with side length 42. First, he places rectangle A, which is 6 times as long as it is wide, inside the square, so that all four vertices of A lie on sides of S, but none of the sides of A are parallel to any side of S. He then places another rectangle B, which is 7 times as long as it is wide, inside rectangle A, so that all four vertices of B lie on sides of A, and again none of the sides of B are parallel to any side of A. Find the length of the shortest side of rectangle B.

p12. Find the value of $\sqrt{3\sqrt{3^3\sqrt{3^5\sqrt{...}}}}$, where the exponents are the odd natural numbers, in increasing order.

p13. Jamesu and Fhomas challenge each other to a game of Square Dance, played on a 9×9 square grid. On Jamesu's turn, he colors in a 2×2 square of uncolored cells pink. On Fhomas's turn, he colors in a 1×1 square of uncolored cells purple. Once Jamesu can no longer make a move, Fhomas gets to color in the rest of the cells purple. If Jamesu goes first, what the maximum number of cells that Fhomas can color purple, assuming both players play optimally in trying to maximize the number of squares of their color?

p14. Triangle ABC is inscribed in circle ω. The tangents to ω from B and C meet at D, and segments AD and BC intersect at E. If $\angle BAC = 60^o$ and the area of $\triangle BDE$ is twice the area of $\triangle CDE$, what is $\frac{AB}{AC}$?

p15. Fhomas and Jamesu are now having a number duel. First, Fhomas chooses a natural number n. Then, starting with Jamesu, each of them take turns making the following moves: if n is composite, the player can pick any prime divisor p of n, and replace n by $n - p$, if n is prime, the player can replace n by $n - 1$. The player who is faced with 1, and hence unable to make a move, loses. How many different numbers $2 \le n \le 2019$ can Fhomas choose such that he has a winning strategy, assuming Jamesu plays optimally?

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2020 **p1.** The number 2020 is very special: the sum of its digits is equal to the product of its nonzero digits. How many such four digit numbers are there? (Numbers with only one nonzero digit, like 3000, also count)

p2. A locker has a combination which is a sequence of three integers between 0 and 49, inclusive. It is known that all of the numbers in the combination are even. Let the total of a lock combination be the sum of the three numbers. Given that the product of the numbers in the combination is 12160, what is the sum of all possible totals of the locker combination?

p3. Given points $A = (0, 0)$ and $B = (0, 1)$ in the plane, the set of all points P in the plane such that triangle ABP is isosceles partitions the plane into k regions. The sum of the areas of those regions that are bounded is s. Find ks.

p4. Three families sit down around a circular table, each person choosing their seat at random. One family has two members, while the other two families have three members. What is the probability that every person sits next to at least one person from a different family?

p5. Jacob and Alexander are walking up an escalator in the airport. Jacob walks twice as fast as Alexander, who takes 18 steps to arrive at the top. Jacob, however, takes 27 steps to arrive at the top. How many of the upward moving escalator steps are visible at any point in time?

p6. Points A, B, C, D, E lie in that order on a circle such that $AB = BC = 5$, $CD = DE = 8$, and $\angle BCD = 150^o$. Let AD and BE intersect at P. Find the area of quadrilateral $PBCD$.

p7. Ivan has a triangle of integers with one number in the first row, two numbers in the second row, and continues up to eight numbers in the eighth row. He starts with the first 8 primes, 2 through 19, in the bottom row. Each subsequent row is filled in by writing the least common multiple of two adjacent numbers in the row directly below. For example, the second last row starts with$6, 15, 35$, etc. Let P be the product of all the numbers in this triangle. Suppose that P is a multiple of a/b, where a and b are positive integers and $a > 1$. Given that b is maximized, and for this value of b, a is also maximized, find $a + b$.

p8. Let $ABCD$ be a cyclic quadrilateral. Given that triangle ABD is equilateral, $\angle CBD = 15^o$, and $AC = 1$, what is the area of $ABCD$?

p9. Let S be the set of all integers greater than 1. The function f is defined on S and each value of f is in S. Given that f is nondecreasing and $f(f(x)) = 2x$ for all x in S, find $f(100)$.

p10. An *origin-symmetric* parallelogram P (that is, if (x, y) is in P, then so is $(-x, -y)$) lies in the coordinate plane. It is given that P has two horizontal sides, with a distance of 2020 between them, and that there is no point with integer coordinates except the origin inside P. Also, P has the maximum possible area satisfying the above conditions. The coordinates of the four vertices of P are $(a, 1010)$, $(b, 1010)$, $(-a, -1010)$, $(-b, -1010)$, where a, b are positive real numbers with $a < b$. What is b?

p11. What is the remainder when $5^{200} + 5^{50} + 2$ is divided by $(5 + 1)(5^2 + 1)(5^4 + 1)$?

p12. Let $f(n) = n^2 - 4096n - 2045$. What is the remainder when $f(f(f(...f(2046)...)))$ is divided by 2047, where the function f is applied 47 times?

p13. What is the largest possible area of a triangle that lies completely within a 97-dimensional hypercube of side length 1, where its vertices are three of the vertices of the hypercube?

p14. Let $N = \left\lfloor \frac{1}{61} \right\rfloor + \left\lfloor \frac{3}{61} \right\rfloor + \left\lfloor \frac{3^2}{61} \right\rfloor + ... + \left\lfloor \frac{3^{2019}}{61} \right\rfloor$. Given that $122N$ can be expressed as $3^a - b$, where a, b are positive integers and a is as large as possible, find $a + b$.

Note: $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x.

p15. Among all ordered triples of integers (x, y, z) that satisfy $x+y+z = 8$ and $x^3+y^3+z^3 = 134$, what is the maximum possible value of $|x| + |y| + |z|$?

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2021 **p1.** Suppose that Yunseo wants to order a pizza that is cut into 4 identical slices. For each slice, there are 2 toppings to choose from: pineapples and apples. Each slice must have exactly one topping. How many distinct pizzas can Yunseo order? Pizzas that can be obtained by rotating one pizza are considered the same.

p2. How many triples of distinct positive integers (E, M, C) are there such that $E = MC^2$ and $E \le 50$?

p3. Given that the cubic polynomial $p(x)$ has leading coefficient 1 and satisfies $p(0) = 0$, $p(1) = 1$, and $p(2) = 2$. Find $p(3)$.

p4. Olaf asks Anna to guess a two-digit number and tells her that it's a multiple of 7 with two distinct digits. Anna makes her first guess. Olaf says one digit is right but in the wrong place. Anna adjusts her guess based on Olaf's comment, but Olaf answers with the same comment again. Anna now knows what the number is. What is the sum of all the numbers that Olaf could have picked?

p5. Vincent the Bug draws all the diagonals of a regular hexagon with area 720, splitting it into many pieces. Compute the area of the smallest piece.

p6. Given that $y - \frac{1}{y} = 7 + \frac{1}{7}$, compute the least integer greater than $y^4 + \frac{1}{y^4}$.

p7. At $9:00$ A.M., Joe sees three clouds in the sky. Each hour afterwards, a new cloud appears in the sky, while each old cloud has a 40% chance of disappearing. Given that the expected number of clouds that Joe will see right after $1:00$ P.M. can be written in the form p/q , where p and q are relatively prime positive integers, what is $p + q$?

p8. Compute the unique three-digit integer with the largest number of divisors.

p9. Jo has a collection of 101 books, which she reads one each evening for 101 evenings in a predetermined order. In the morning of each day that Jo reads a book, Amy chooses a random book from Jo's collection and burns one page in it. What is the expected number of pages that Jo misses?

p10. Given that x, y, z are positive real numbers satisfying $2x + y = 14 - xy$, $3y + 2z = 30 - yz$, and $z + 3x = 69 - zx$, the expression $x + y + z$ can be written as $p\sqrt{q} - r$, where p, q, r are positive integers and q is not divisible by the square of any prime. Compute $p + q + r$.

p11. In rectangle $TRIG$, points A and L lie on sides TG and TR respectively such that $TA = AG$ and $TL = 2LR$. Diagonal GR intersects segments IL and IA at B and E respectively. Suppose that the area of the convex pentagon with vertices $TABLE$ is equal to 21. What is the area of $TRIG$?

p12. Call a number nice if it can be written in the form $2^m \cdot 3^n$, where m and n are nonnegative integers. Vincent the Bug fills in a 3 by 3 grid with distinct nice numbers, such that the product of the numbers in each row and each column are the same. What is the smallest possible value of the largest number Vincent wrote?

p13. Let $s(n)$ denote the sum of digits of positive integer n and define $f(n) = s(202n) - s(22n)$.

Given that M is the greatest possible value of $f(n)$ for $0 < n < 350$ and N is the least value such that $f(N) = M$, compute $M + N$.

p14. In triangle ABC, let M be the midpoint of BC and let E, F be points on AB, AC, respectively, such that $\angle MEF = 30^o$ and $\angle MFE = 60^o$. Given that $\angle A = 60^o$, $AE = 10$, and $EB = 6$,compute $AB + AC$.

p15. A unit cube moves on top of a 6×6 checkerboard whose squares are unit squares. Beginning in the bottom left corner, the cube is allowed to roll up or right, rolling about its bottom edges to travel from square to square, until it reaches the top right corner. Given that the side of the cube facing upwards in the beginning is also facing upwards after the cube reaches the top right corner, how many total paths are possible?

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2022 **p1.** Compute $1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55$.

p2. Given that a, b, and c are positive integers such that $a + b = 9$ and $bc = 30$, find the minimum possible value of $a + c$.

p3. Points X and Y lie outside regular pentagon $ABCDE$ such that ABX and DEY are equilateral triangles. Find the degree measure of $\angle XCY$.

p4. Let N be the product of the positive integer divisors of $8!$, including itself. The largest integer power of 2 that divides N is 2^k. Compute k.

p5. Let $A = (-20, 22)$, $B = (k, 0)$, and $C = (202, 2)$ be points on the coordinate plane. Given that $\angle ABC = 90^o$, find the sum of all possible values of k.

p6. Tej is typing a string of Ls and Os that consists of exactly 7 Ls and 4 Os. How many different strings can he type that do not contain the substring 'LOL' anywhere? A substring is a sequence of consecutive letters contained within the original string.

p7. How many ordered triples of integers (a, b, c) satisfy both $a+b-c = 12$ and $a^2 +b^2 -c^2 = 24$?

p8. For how many three-digit base-7 numbers $\overline{ABC}_7$ does $\overline{ABC}_7$ divide $\overline{ABC}_{10}$? (Note: $\overline{ABC}_D$

refers to the number whose digits in base D are, from left to right, A, B, and C; for example, $\overline{123}_4$ equals 27 in base ten).

p9. Natasha is sitting on one of the 35 squares of a 5-by-7 grid of squares. Wanda wants to walk through every square on the board exactly once except the one Natasha is on, starting and ending on any 2 squares she chooses, such that from any square she can only go to an adjacent square (two squares are adjacent if they share an edge). How many squares can Natasha choose to sit on such that Wanda cannot go on her walk?

p10. In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Point P lies inside ABC and points D, E, and F lie on sides BC, CA, and AB, respectively, so that $PD \perp BC$, $PE \perp CA$, and $PF \perp AB$. Given that PD, PE, and PF are all integers, find the sum of all possible distinct values of $PD \cdot PE \cdot PF$.

p11. A palindrome is a positive integer which is the same when read forwards or backwards. Find the sum of the two smallest palindromes that are multiples of 137.

p12. Let $P(x) = x^2+px+q$ be a quadratic polynomial with positive integer coefficients. Compute the least possible value of p such that 220 divides p and the equation $P(x^3) = P(x)$ has at least four distinct integer solutions.

p13. Everyone at a math club is either a truth-teller, a liar, or a piggybacker. A truth-teller always tells the truth, a liar always lies, and a piggybacker will answer in the style of the previous person who spoke (i.e., if the person before told the truth, they will tell the truth, and if the person before lied, then they will lie). If a piggybacker is the first one to talk, they will randomly either tell the truth or lie. Four seniors in the math club were interviewed and here was their conversation:

Neil: There are two liars among us.
Lucy: Neil is a piggybacker.
Kevin: Excluding me, there are more truth-tellers than liars here.
Neil: Actually, there are more liars than truth-tellers if we exclude Kevin.
Jacob: One plus one equals three.

Define the base-4 number $M = \overline{NLKJ}_4$, where each digit is 1 for a truth-teller, 2 for a piggybacker, and 3 for a liar (N corresponds to Neil, L to Lucy, K corresponds to Kevin, and J corresponds to Jacob). What is the sum of all possible values of M, expressed in base 10?

p14. An equilateral triangle of side length 8 is tiled by 64 equilateral triangles of unit side length to form a triangular grid. Initially, each triangular cell is either living or dead. The grid evolves over time under the following rule: every minute, if a dead cell is edge-adjacent to at least two living cells, then that cell becomes living, and any living cell remains living. Given that every cell in the grid eventually evolves to be living, what is the minimum possible number of living cells in the initial grid?

p15. In triangle ABC, $AB = 7$, $BC = 11$, and $CA = 13$. Let Γ be the circumcircle of ABC and let M, N, and P be the midpoints of minor arcs BC , CA, and AB of Γ, respectively. Given that K denotes the area of ABC and L denotes the area of the intersection of ABC and MNP, the ratio L/K can be written as a/b , where a and b are relatively prime positive integers. Compute $a + b$.

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2023 **p1.** We define $a \oplus b = \frac{ab}{a+b}$. Compute $(3 \oplus 5) \oplus (5 \oplus 4)$.

p2. Let $ABCD$ be a quadrilateral with $\angle A = 45^o$ and $\angle B = 45^o$. If $BC = 5\sqrt{2}$, $AD = 6\sqrt{2}$, and $AB = 18$, find the length of side CD.

p3. A positive real number x satisfies the equation $x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2} = 10$. Find the sum of all possible values of $x + 1 + \frac{1}{x}$.

p4. David writes 6 positive integers on the board (not necessarily distinct) from least to greatest. The mean of the first three numbers is 3, the median of the first four numbers is 4, the unique mode of the first five numbers is 5, and the range of all 6 numbers is 6. Find the maximum possible value of the product of David's 6 integers.

p5. Let $ABCD$ be a convex quadrilateral such that $\angle A = \angle B = 120^o$ and $\angle C = \angle D = 60^o$. There exists a circle with center I which is tangent to all four sides of $ABCD$. If $IA \cdot IB \cdot IC \cdot ID = 240$, find the area of quadrilateral $ABCD$.

p6. The letters $EXETERMATH$ are placed into cells on an annulus as shown below. How many ways are there to color each cell of the annulus with red, blue, green, or yellow such that each letter is always colored the same color and adjacent cells are always colored differently?
`https://cdn.artofproblemsolving.com/attachments/3/5/b470a771a5279a7746c06996f2bb5487c33ec
png`

p7. Let $ABCD$ be a square, and let ω be a quarter circle centered at A passing through points B and D. Points E and F lie on sides BC and CD respectively. Line EF intersects ω at two points, G and H. Given that $EG = 2$, $GH = 16$ and $HF = 9$, find the length of side AB.

p8. Let x be equal to $\frac{2022!+2021!}{2020!+2019!+2018!}$. Find the closest integer to $2\sqrt{x}$.

p9. For how many ordered pairs of positive integers (m, n) is the absolute difference between $lcm(m, n)$ and $gcd(m, n)$ equal to 2023?

p10. There are 2023 distinguishable frogs sitting on a number line with one frog sitting on i for all integers i between -1011 and 1011, inclusive. Each minute, every frog randomly jumps either one unit left or one unit right with equal probability. After 1011 minutes, over all possible arrangements of the frogs, what is the average number of frogs sitting on the number 0?

p11. Albert has a calculator initially displaying 0 with two buttons: the first button increases the number on the display by one, and the second button returns the square root of the number on the display. Each second, he presses one of the two buttons at random with equal probability. What is the probability that Albert's calculator will display the number 6 at some point?

p12. For a positive integer $k \geq 2$, let $f(k)$ be the number of positive integers n such that n divides $(n-1)!+k$. Find

$$f(2) + f(3) + f(4) + f(5) + ... + f(100).$$

p13. Mr. Atf has nine towers shaped like rectangular prisms. Each tower has a 1 by 1 base. The first tower as height 1, the next has height 2, up until the ninth tower, which has height 9. Mr. Atf randomly arranges these 9 towers on his table so that their square bases form a 3 by 3 square on the surface of his table. Over all possible solids Mr. Atf could make, what is the average surface area of the solid?

p14. Let $ABCD$ be a cyclic quadrilateral whose diagonals are perpendicular. Let E be the intersection of AC and BD, and let the feet of the altitudes from E to the sides AB, BC, CD, DA be W, X, Y, Z respectively. Given that $EW = 2EY$ and $EW \cdot EX \cdot EY \cdot EZ = 36$, find the minimum possible value of $\frac{1}{[EAB]} + \frac{1}{[EBC]} + \frac{1}{[ECD]} + \frac{1}{[EDA]}$. The notation $[XYZ]$ denotes the area of triangle XYZ.

p15. Given that $x^2 - xy + y^2 = (x + y)^3$, $y^2 - yz + z^2 = (y + z)^3$, and $z^2 - zx + x^2 = (z + x)^3$ for complex numbers x, y, z, find the product of all distinct possible nonzero values of $x + y + z$.

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2024 **p1.** Warren interrogates the 25 members of his cabinet, each of whom always lies or always tells the truth. He asks them all, "How many of you always lie?" He receives every integer answer from 1 to 25 exactly once. Find the actual number of liars in his cabinet.

p2. Abraham thinks of distinct nonzero digits E, M, and C such that $E + M = \overline{CC}$.
Help him evaluate the sum of the two digit numbers $\overline{EC}$ and $\overline{MC}$. (Note that $\overline{CC}$, $\overline{EC}$, and $\overline{MC}$ are read as two-digit numbers.)

p3. Let ω, Ω, Γ be concentric circles such that Γ is inside Ω and Ω is inside ω. Points A, B, C on ω and D, E on Ω are chosen such that line AB is tangent to Ω, line AC is tangent to Γ, and line DE is tangent to Γ. If $AB = 21$ and $AC = 29$, find DE.

p4. Let a, b, and c be three prime numbers such that $a + b = c$. If the average of two of the three primes is four less than four times the fourth power of the last, find the second-largest of the three primes.

p5. At Stillwells Ice Cream, customers must choose one type of scoop and two different types of toppings. There are currently 630 different combinations a customer could order. If another topping is added to the menu, there would be 840 different combinations. If, instead, another type of scoop were added to the menu, compute the number of different combinations there would be.

p6. Eleanor the ant takes a path from $(0, 0)$ to $(20, 24)$, traveling either one unit right or one unit up each second. She records every lattice point she passes through, including the starting and ending point. If the sum of all the x-coordinates she records is 271, compute the sum of all the y-coordinates. (A lattice point is a point with integer coordinates.)

p7. Teddy owns a square patch of desert. He builds a dam in a straight line across the square, splitting the square into two trapezoids. The perimeters of the trapezoids are64 miles and 76 miles, and their areas differ by 135 square miles. Find, in miles, the length of the segment that divides them.

p8. Michelle is playing Spot-It with a magical deck of 10 cards. Each card has 10 distinct symbols on it, and every pair of cards shares exactly 1 symbol. Find the minimum number of distinct symbols on all of the cards in total.

p9. Define the function $f(n) = \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} + ...$ for integers $n \ge 2$. Find

$$f(2) + f(4) + f(6) + ...$$

p10. There are 9 indistinguishable ants standing on a 3×3 square grid. Each ant is standing on exactly one square. Compute the number of different ways the ants can stand so that no column or row contains more than 3 ants.

p11. Let $s(N)$ denote the sum of the digits of N. Compute the sum of all two-digit positive integers N for which $s(N^2) = s(N)^2$.

p12. Martha has two square sheets of paper, A and B. With each sheet, she repeats the following process four times: fold bottom side to top side, fold right side to left side. With sheet A, she then makes a cut from the top left corner to the bottom right. With sheet B, she makes a cut from the bottom left corner to the top right. Find the total number of pieces of paper yielded from sheets A and sheets B.
https://cdn.artofproblemsolving.com/attachments/f/6/ff3a459a135562002aa2c95067f3f01441d62 png

p13. Let x and y be positive integers such that gcd $(x^y, y^x) = 2^{28}$. Find the sum of all possible values of min (x, y).

p14. Convex hexagon $TRUMAN$ has opposite sides parallel. If each side has length 3 and the area of this hexagon is 5, compute

$$TU \cdot RM \cdot UA \cdot MN \cdot AT \cdot NR.$$

p15. Let x, y, and z be positive real numbers satisfying the system

$$\begin{cases} x^2 + xy + y^2 = 25 \\ y^2 + yz + z^2 = 36 \\ z^2 + zx + x^2 = 49 \end{cases}$$

Compute $x^2 + y^2 + z^2$.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).