Berkley mini Math Tournament Fall 2016

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by parmenides51

Team Round **p1.** BmMT is in a week, and we don't have any problems! Let's write 1 on the first day, 2 on the second day, 4 on the third, 8 on the fourth, 16 on the fifth, 32 on the sixth, and 64 on the seventh. After seven days, how many problems will we have written in total?

p2. 100 students are taking a ten-point exam. 50 students scored 8 points, 30 students scored 7 points, and the rest scored 9 points. What is the average score for the exam?

p3. Rebecca has four pairs of shoes. Rebecca may or may not wear matching shoes. However, she will always use a left-shoe for her left foot and a right-shoe for her right foot. How many ways can Rebecca wear shoes?

p4. A council of 111 mathematicians voted on whether to hold their conference in Beijing or Shanghai. The outcome of an initial vote was 70 votes in favor of Beijing, and 41 votes in favor of Shanghai. If the vote were to be held again, what is the minimum number of mathematicians that would have to change their votes in order for Shanghai to win a majority of votes?

p5. What is the area of the triangle bounded by the line $20x + 16y = 160$, the x-axis, and the y-axis?

p6. Suppose that 3 runners start running from the start line around a circular 800-meter track and that their speeds are 100, 160, and 200 meters per minute, respectively. How many minutes will they run before all three are next at the start line at the same time?

p7. Brian's lawn is in the shape of a circle, with radius 10 meters. Brian can throw a frisbee up to 50 meters from where he stands. What is the area of the region (in square meters) in which the frisbee can land, if Brian can stand anywhere on his lawn?

p8. A seven digit number is called "bad" if exactly four of its digits are 0 and the rest are odd. How many seven digit numbers are bad?

p9. Suppose you have a 3-digit number with only even digits. What is the probability that twice that number also has only even digits?

p10. You have a flight on Air China from Beijing to New York. The flight will depart any time between 1 p.m. and 6 p.m., uniformly at random. Your friend, Henry, is flying American Airlines, also from Beijing to New York. Henry's flight will depart any time between 3 p.m. and 5 p.m., uniformly at random. What is the probability that Henry's flight departs before your flight?

p11. In the figure below, three semicircles are drawn outside the given right triangle. Given the areas $A_1 = 17$ and $A_2 = 14$, find the area A_3.
https://cdn.artofproblemsolving.com/attachments/4/4/28393acb3eba83a5a489e14b30a3e84ffa60f
png

p12. Consider a circle of radius 1 drawn tangent to the positive x and y axes. Now consider another smaller circle tangent to that circle and also tangent to the positive x and y axes. Find the radius of the smaller circle.
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p13. The following expression is an integer. Find this integer: $\frac{\sqrt{20+16\frac{\sqrt{20+16\frac{20+16...}{2}}}{2}}}{2}$

p14. Let $2016 = a_1 \times a_2 \times ... \times a_n$ for some positive integers $a_1, a_2, ..., a_n$. Compute the smallest possible value of $a_1 + a_2 + ... + a_n$.

p15. The tetranacci numbers are defined by the recurrence $T_n = T_{n-1} + T_{n-2} + T_{n-3} + T_{n-4}$ and $T_0 = T_1 = T_2 = 0$ and $T_3 = 1$. Given that $T_9 = 29$ and $T_{14} = 773$, calculate T_{15}.

p16. Find the number of zeros at the end of $(2016!)^{2016}$. Your answer should be an integer, not its prime factorization.

p17. A DJ has 7 songs named $1, 2, 3, 4, 5, 6$, and 7. He decides that no two even-numbered songs can be played one after the other. In how many different orders can the DJ play the 7 songs?

p18. Given a cube, how many distinct ways are there (using 6 colors) to color each face a distinct color? Colorings are distinct if they cannot be transformed into one another by a sequence of rotations.

p19. Suppose you have a triangle with side lengths $3, 4$, and 5. For each of the triangle's sides, draw a square on its outside. Connect the adjacent vertices in order, forming 3 new triangles (as in the diagram). What is the area of this convex region?

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p20. Find x such that $\sqrt{c+\sqrt{c-x}} = x$ when $c = 4$.

PS. You had better use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

Ind. Round **p1.** David is taking a 50-question test, and he needs to answer at least 70% of the questions correctly in order to pass the test. What is the minimum number of questions he must answer correctly in order to pass the test?

p2. You decide to flip a coin some number of times, and record each of the results. You stop flipping the coin once you have recorded either 20 heads, or 16 tails. What is the maximum number of times that you could have flipped the coin?

p3. The width of a rectangle is half of its length. Its area is 98 square meters. What is the length of the rectangle, in meters?

p4. Carol is twice as old as her younger brother, and Carol's mother is 4 times as old as Carol is. The total age of all three of them is 55. How old is Carol's mother?

p5. What is the sum of all two-digit multiples of 9?

p6. The number 2016 is divisible by its last two digits, meaning that 2016 is divisible by 16. What is the smallest integer larger than 2016 that is also divisible by its last two digits?

p7. Let Q and R both be squares whose perimeters add to 80. The area of Q to the area of R is in a ratio of $16 : 1$. Find the side length of Q.

p8. How many 8-digit positive integers have the property that the digits are strictly increasing from left to right? For instance, 12356789 is an example of such a number, while 12337889 is not.

p9. During a game, Steve Korry attempts 20 free throws, making 16 of them. How many more free throws does he have to attempt to finish the game with 84% accuracy, assuming he makes them all?

p10. How many di erent ways are there to arrange the letters $MILKTEA$ such that TEA is a contiguous substring?
For reference, the term "contiguous substring" means that the letters TEA appear in that order, all next to one another. For example, $MITEALK$ would be such a string, while $TMIELKA$ would not be.

p11. Suppose you roll two fair 20-sided dice. What is the probability that their sum is divisible by 10?

p12. Suppose that two of the three sides of an acute triangle have lengths 20 and 16, respectively. How many possible integer values are there for the length of the third side?

p13. Suppose that between Beijing and Shanghai, an airplane travels 500 miles per hour, while a train travels at 300 miles per hour. You must leave for the airport 2 hours before your flight, and must leave for the train station 30 minutes before your train. Suppose that the two methods of transportation will take the same amount of time in total. What is the distance, in miles, between the two cities?

p14. How many nondegenerate triangles (triangles where the three vertices are not collinear) with integer side lengths have a perimeter of 16? Two triangles are considered distinct if they are not congruent.

p15. John can drive 100 miles per hour on a paved road and 30 miles per hour on a gravel road. If it takes John 100 minutes to drive a road that is 100 miles long, what fraction of the time does John spend on the paved road?

p16. Alice rolls one pair of 6-sided dice, and Bob rolls another pair of 6-sided dice. What is the probability that at least one of Alice's dice shows the same number as at least one of Bob's dice?

p17. When 20^{16} is divided by 16^{20} and expressed in decimal form, what is the number of digits to the right of the decimal point? Trailing zeroes should not be included.

p18. Suppose you have a 20×16 bar of chocolate squares. You want to break the bar into smaller chunks, so that after some sequence of breaks, no piece has an area of more than 5. What is the minimum possible number of times that you must break the bar?
For an example of how breaking the chocolate works, suppose we have a 2×2 bar and wish to

break it entirely into 1×1 bars. We can break it once to get two 2×1 bars. Then, we would have to break each of these individual bars in half in order to get all the bars to be size 1×1, and we end up using 3 breaks in total.

p19. A class of 10 students decides to form two distinguishable committees, each with 3 students. In how many ways can they do this, if the two committees can have no more than one student in common?

p20. You have been told that you are allowed to draw a convex polygon in the Cartesian plane, with the requirements that each of the vertices has integer coordinates whose values range from 0 to 10 inclusive, and that no pair of vertices can share the same x or y coordinate value (so for example, you could not use both $(1, 2)$ and $(1, 4)$ in your polygon, but $(1, 2)$ and $(2, 1)$ is fine). What is the largest possible area that your polygon can have?

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