Berkley mini Math Tournament Fall 2017
www.artofproblemsolving.com/community/c2826290
by parmenides51

Team Round **p1.** Suppose $a_1 \cdot 2 = a_2 \cdot 3 = a_3$ and $a_1 + a_2 + a_3 = 66$. What is a_3?

p2. Ankit buys a see-through plastic cylindrical water bottle. However, in coming home, he accidentally hits the bottle against a wall and dents the top portion of the bottle (above the 7 cm mark). Ankit now wants to determine the volume of the bottle. The area of the base of the bottle is 20 cm^2 . He fills the bottle with water up to the 5 cm mark. After flipping the bottle upside down, he notices that the height of the empty space is at the 7 cm mark. Find the total volume (in cm^3) of this bottle.
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png

p3. If P is a quadratic polynomial with leading coefficient 1 such that $P(1) = 1$, $P(2) = 2$, what is $P(10)$?

p4. Let ABC be a triangle with $AB = 1$, $AC = 3$, and $BC = 3$. Let D be a point on BC such that $BD = \frac13$. What is the ratio of the area of BAD to the area of CAD?

p5. A coin is flipped 12 times. What is the probability that the total number of heads equals the total number of tails? Express your answer as a common fraction in lowest terms.

p6. Moor pours 3 ounces of ginger ale and 1 ounce of lime juice in cup A, 3 ounces of lime juice and 1 ounce of ginger ale in cup B, and mixes each cup well. Then he pours 1 ounce of cup A into cup B, mixes it well, and pours 1 ounce of cup B into cup A. What proportion of cup A is now ginger ale? Express your answer as a common fraction in lowest terms.

p7. Determine the maximum possible area of a right triangle with hypotenuse 7. Express your answer as a common fraction in lowest terms.

p8. Debbie has six Pusheens: 2 pink ones, 2 gray ones, and 2 blue ones, where Pusheens of the same color are indistinguishable. She sells two Pusheens each to Alice, Bob, and Eve. How many ways are there for her to do so?

p9. How many nonnegative integer pairs (a, b) are there that satisfy $ab = 90 - a - b$?

p10. What is the smallest positive integer $a_1...a_n$ (where $a_1, ..., a_n$ are its digits) such that $9 \cdot a_1...a_n = a_n...a_1$, where $a_1, a_n \neq 0$?

p11. Justin is growing three types of Japanese vegetables: wasabi root, daikon and matsutake mushrooms. Wasabi root needs 2 square meters of land and 4 gallons of spring water to grow, matsutake mushrooms need 3 square meters of land and 3 gallons of spring water, and daikon need 1 square meter of land and 1 gallon of spring water to grow. Wasabi sell for 60 per root, matsutake mushrooms sell for 60 per mushroom, and daikon sell for 2 per root. If Justin has 500 gallons of spring water and 400 square meters of land, what is the maximum amount of money, in dollars, he can make?

p12. A *prim* number is a number that is prime if its last digit is removed. A *rime* number is a number that is prime if its first digit is removed. Determine how many numbers between 100 and 999 inclusive are both prim and rime numbers.

p13. Consider a cube. Each corner is the intersection of three edges; slice off each of these corners through the midpoints of the edges, obtaining the shape below. If we start with a $2 \times 2 \times 2$ cube, what is the volume of the resulting solid?
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png

p14. If a parallelogram with perimeter 14 and area 12 is inscribed in a circle, what is the radius of the circle?

p15. Take a square $ABCD$ of side length 1, and draw $\overline{AC}$. Point E lies on $\overline{BC}$ such that $\overline{AE}$ bisects $\angle BAC$. What is the length of BE?

p16. How many integer solutions does $f(x) = (x^2 + 1)(x^2 + 2) + (x^2 + 3)(x + 4) = 2017$ have?

p17. Alice, Bob, Carol, and Dave stand in a circle. Simultaneously, each player selects another player at random and points at that person, who must then sit down. What is the probability that Alice is the only person who remains standing?

p18. Let x be a positive integer with a remainder of 2 when divided by 3, 3 when divided by 4, 4 when divided by 5, and 5 when divided by 6. What is the smallest possible such x?

p19. A circle is inscribed in an isosceles trapezoid such that all four sides of the trapezoid are tangent to the circle. If the radius of the circle is 1, and the upper base of the trapezoid is 1, what is the area of the trapezoid?

p20. Ray is blindfolded and standing 1 step away from an ice cream stand. Every second, he has a $1/4$ probability of walking 1 step towards the ice cream stand, and a $3/4$ probability of walking 1 step away from the ice cream stand. When he is 0 steps away from the ice cream stand, he wins. What is the probability that Ray eventually wins?

PS. You had better use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

Ind. Round **p1.** It's currently $6 : 00$ on a 12 hour clock. What time will be shown on the clock 100 hours from now? Express your answer in the form hh : mm.

p2. A tub originally contains 10 gallons of water. Alex adds some water, increasing the amount of water by 20

p3. There are 2000 math students and 4000 CS students at Berkeley. If 5580 students are either math students or CS students, then how many of them are studying both math and CS?

p4. Determine the smallest integer x greater than 1 such that x^2 is one more than a multiple of 7.

p5. Find two positive integers x, y greater than 1 whose product equals the following sum:

$$9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29.$$

Express your answer as an ordered pair (x, y) with $x \le y$.

p6. The average walking speed of a cow is 5 meters per hour. If it takes the cow an entire day to walk around the edges of a perfect square, then determine the area (in square meters) of this square.

p7. Consider the cube below. If the length of the diagonal AB is $3\sqrt{3}$, determine the volume of the cube.
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`png`

p8. I have 18 socks in my drawer, 6 colored red, 8 colored blue and 4 colored green. If I close my eyes and grab a bunch of socks, how many socks must I grab to guarantee there will be two pairs of matching socks?

p9. Define the operation $a@b$ to be $3 + ab + a + 2b$. There exists a number x such that $x@b = 1$ for all b. Find x.

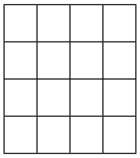
p10. Compute the units digit of $2017^{(2017^2)}$.

p11. The distinct rational numbers $-\sqrt{-x}$, x, and $-x$ form an arithmetic sequence in that order. Determine the value of x.

p12. Let $y = x^2 + bx + c$ be a quadratic function that has only one root. If b is positive, find $\frac{b+2}{\sqrt{c}+1}$.

p13. Alice, Bob, and four other people sit themselves around a circular table. What is the probability that Alice does not sit to the left or right of Bob?

p14. Let $f(x) = |x - 8|$. Let p be the sum of all the values of x such that $f(f(f(x))) = 2$ and q be the minimum solution to $f(f(f(x))) = 2$. Compute $p \cdot q$.

p15. Determine the total number of rectangles (1×1, 1×2, 2×2, etc.) formed by the lines in the figure below: 

p16. Take a square $ABCD$ of side length 1, and let P be the midpoint of AB. Fold the square so that point D touches P, and let the intersection of the bottom edge DC with the right edge be Q. What is BQ?
`https://cdn.artofproblemsolving.com/attachments/1/1/aeed2c501e34a40a8a786f6bb60922b614a36`
`png`

p17. Let A, B, and k be integers, where k is positive and the greatest common divisor of A, B, and k is 1. Define $x\#y$ by the formula $x\#y = \frac{Ax+By}{kxy}$. If $8\#4 = \frac{1}{2}$ and $3\#1 = \frac{13}{6}$, determine the sum $A + B + k$.

p18. There are 20 indistinguishable balls to be placed into bins A, B, C, D, and E. Each bin must have at least 2 balls inside of it. How many ways can the balls be placed into the bins, if each ball must be placed in a bin?

p19. Let T_i be a sequence of equilateral triangles such that
(a) T_1 is an equilateral triangle with side length 1.
(b) T_{i+1} is inscribed in the circle inscribed in triangle T_i for $i \ge 1$.
Find
$$\sum_{i=1}^{\infty} Area(T_i).$$

p20. A *gorgeous* sequence is a sequence of 1's and 0's such that there are no consecutive 1's. For instance, the set of all gorgeous sequences of length 3 is $\{[1, 0, 0]$,$[1, 0, 1]$, $[0, 1, 0]$, $[0, 0, 1]$, $[0, 0, 0]\}$. Determine the number of gorgeous sequences of length 7.

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Ind. Tie **p1.** Consider a 4×4 lattice on the coordinate plane. At $(0, 0)$ is Mori's house, and at $(4, 4)$ is Mori's workplace. Every morning, Mori goes to work by choosing a path going up and right along the roads on the lattice. Recently, the intersection at $(2, 2)$ was closed. How many ways are there now for Mori to go to work?

p2. Given two integers, define an operation $*$ such that if a and b are integers, then a $*$ b is an integer. The operation $*$ has the following properties:
1. $a * a$ = 0 for all integers a.
2. $(ka + b) * a = b * a$ for integers a, b, k.
3. $0 \le b * a < a$.
4. If $0 \le b < a$, then $b * a = b$.
Find $2017 * 16$.

p3. Let ABC be a triangle with side lengths $AB = 13$, $BC = 14$, $CA = 15$. Let A', B', C', be the midpoints of BC, CA, and AB, respectively. What is the ratio of the area of triangle ABC to the area of triangle $A'B'C'$?

p4. In a strange world, each orange has a label, a number from 0 to 10 inclusive, and there are an infinite number of oranges of each label. Oranges with the same label are considered indistinguishable. Sally has 3 boxes, and randomly puts oranges in her boxes such that
(a) If she puts an orange labelled a in a box (where a is any number from 0 to 10), she cannot put any other oranges labelled a in that box.
(b) If any two boxes contain an orange that have the same labelling, the third box must also contain an orange with that labelling.
(c) The three boxes collectively contain all types of oranges (oranges of any label).

The number of possible ways Sally can put oranges in her 3 boxes is N, which can be written as the product of primes:

$$p_1^{e_1}p_2^{e_2}...p_k^{e_k}$$

where $p_1 \ne p_2 \ne p_3... \ne p_k$ and p_i are all primes and e_i are all positive integers. What is the sum $e_1 + e_2 + e_3 + ... + e_k$?

p5. Suppose I want to stack 2017 identical boxes. After placing the first box, every subsequent box must either be placed on top of another one or begin a new stack to the right of the rightmost pile. How many different ways can I stack the boxes, if the order I stack them doesn't matter? Express your answer as

$$p_1^{e_1}p_2^{e_2}...p_n^{e_n}$$

where $p_1, p_2, p_3, ..., p_n$ are distinct primes and e_i are all positive integers.

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