Berkley mini Math Tournament Fall 2018
www.artofproblemsolving.com/community/c2826291
by parmenides51

Team Round **p1.** What is the sum of the first 12 positive integers?

p2. How many positive integers less than or equal to 100 are multiples of both 2 and 5?

p3. Alex has a bag with 4 white marbles and 4 black marbles. She takes 2 marbles from the bag without replacement. What is the probability that both marbles she took are black? Express your answer as a decimal or a fraction in lowest terms.

p4. How many 5-digit numbers are there where each digit is either 1 or 2?

p5. An integer a with $1 \le a \le 10$ is randomly selected. What is the probability that $\frac{100}{a}$ is an integer? Express your answer as decimal or a fraction in lowest terms.

p6. Two distinct non-tangent circles are drawn so that they intersect each other. A third circle, distinct from the previous two, is drawn. Let P be the number of points of intersection between any two circles. How many possible values of P are there?

p7. Let x, y, z be nonzero real numbers such that $x + y + z = xyz$. Compute

$$\frac{1 + yz}{yz} + \frac{1 + xz}{xz} + \frac{1 + xy}{xy}.$$

p8. How many positive integers less than 106 are simultaneously perfect squares, cubes, and fourth powers?

p9. Let C_1 and C_2 be two circles centered at point O of radii 1 and 2, respectively. Let A be a point on C_2. We draw the two lines tangent to C_1 that pass through A, and label their other intersections with C_2 as B and C. Let x be the length of minor arc BC, as shown. Compute x.
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`png`

p10. A circle of area π is inscribed in an equilateral triangle. Find the area of the triangle.

p11. Julie runs a 2 mile route every morning. She notices that if she jogs the route 2 miles per hour faster than normal, then she will finish the route 5 minutes faster. How fast (in miles per hour) does she normally jog?

p12. Let $ABCD$ be a square of side length 10. Let $EFGH$ be a square of side length 15 such that E is the center of $ABCD$, EF intersects BC at X, and EH intersects CD at Y (shown below). If $BX = 7$, what is the area of quadrilateral $EXCY$?
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png

p13. How many solutions are there to the system of equations

$$a^2 + b^2 = c^2$$

$$(a + 1)^2 + (b + 1)^2 = (c + 1)^2$$

if a, b, and c are positive integers?

p14. A square of side length s is inscribed in a semicircle of radius r as shown. Compute $\frac{s}{r}$.
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p15. S is a collection of integers n with $1 \le n \le 50$ so that each integer in S is composite and relatively prime to every other integer in S. What is the largest possible number of integers in S?

p16. Let $ABCD$ be a regular tetrahedron and let W, X, Y, Z denote the centers of faces ABC, BCD, CDA, and DAB, respectively. What is the ratio of the volumes of tetrahedrons $WXYZ$ and $WAYZ$? Express your answer as a decimal or a fraction in lowest terms.

p17. Consider a random permutation $\{s_1, s_2, ..., s_8\}$ of $\{1, 1, 1, 1, -1, -1, -1, -1\}$. Let S be the largest of the numbers s_1, $s_1 + s_2$, $s_1 + s_2 + s_3$, ... , $s_1 + s_2 + ... + s_8$. What is the probability that S is exactly 3? Express your answer as a decimal or a fraction in lowest terms.

p18. A positive integer is called *almost-kinda-semi-prime* if it has a prime number of positive integer divisors. Given that there $are 168$ primes less than 1000, how many almost-kinda-semi-prime numbers are there less than 1000?

p19. Let $ABCD$ be a unit square and let X, Y, Z be points on sides AB, BC, CD, respectively,

such that $AX = BY = CZ$. If the area of triangle XYZ is $\frac{1}{3}$, what is the maximum value of the ratio XB/AX?
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p20. Positive integers $a \le b \le c$ have the property that each of $a + b$, $b + c$, and $c + a$ are prime. If $a + b + c$ has exactly 4 positive divisors, find the fourth smallest possible value of the product $c(c + b)(c + b + a)$.

PS. You had better use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

Ind. Round **p1.** If x is a real number that satisfies $\frac{48}{x} = 16$, find the value of x.

p2. If ABC is a right triangle with hypotenuse BC such that $\angle ABC = 35^o$, what is $\angle BCA$ in degrees?
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png

p3. If $a \vartriangle b = a + b - ab$, find $4 \vartriangle 9$.

p4. Grizzly is 6 feet tall. He measures his shadow to be 4 feet long. At the same time, his friend Panda helps him measure the shadow of a nearby lamp post, and it is 6 feet long. How tall is the lamp post in feet?

p5. Jerry is currently twice as old as Tom was 7 years ago. Tom is 6 years younger than Jerry. How many years old is Tom?

p6. Out of the $10,000$ possible four-digit passcodes on a phone, how many of them contain only prime digits?

p7. It started snowing, which means Moor needs to buy snow shoes for his 6 cows and 7 sky bison. A cow has 4 legs, and a sky bison has 6 legs. If Moor has 36 snow shoes already, how many more shoes does he need to buy? Assume cows and sky bison wear the same type of shoe and each leg gets one shoe.

p8. How many integers n with $1 \le n \le 100$ have exactly 3 positive divisors?

p9. James has three 3 candies and 3 green candies. 3 people come in and each randomly take 2 candies. What is the probability that no one got 2 candies of the same color? Express your answer as a decimal or a fraction in lowest terms.

p10. When Box flips a strange coin, the coin can land heads, tails, or on the side. It has a $\frac{1}{10}$probability of landing on the side, and the probability of landing heads equals the probability of landing tails. If Box flips a strange coin 3 times, what is the probability that the number of heads flipped is equal to the number of tails flipped? Express your answer as a decimal or a fraction in lowest terms.

p11. James is travelling on a river. His canoe goes 4 miles per hour upstream and 6 miles per hour downstream. He travels 8 miles upstream and then 8 miles downstream (to where he started). What is his average speed, in miles per hour? Express your answer as a decimal or a fraction in lowest terms.

p12. Four boxes of cookies and one bag of chips cost exactly 1000 jelly beans. Five bags of chips and one box of cookies cost less than 1000 jelly beans. If both chips and cookies cost a whole number of jelly beans, what is the maximum possible cost of a bag of chips?

p13. June is making a pumpkin pie, which takes the shape of a truncated cone, as shown below. The pie tin is 18 inches wide at the top, 16 inches wide at the bottom, and 1 inch high. How many cubic inches of pumpkin filling are needed to fill the pie?
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p14. For two real numbers a and b, let $a\#b = ab - 2a - 2b + 6$. Find a positive real number x such that $(x\#7)\#x = 82$.

p15. Find the sum of all positive integers n such that $\frac{n^2+20n+51}{n^2+4n+3}$ is an integer.

p16. Let ABC be a right triangle with hypotenuse AB such that $AC = 36$ and $BC = 15$. A semicircle is inscribed in ABC as shown, such that the diameter XC of the semicircle lies on side AC and that the semicircle is tangent to AB. What is the radius of the semicircle?
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p17. Let a and b be relatively prime positive integers such that the product ab is equal to the least common multiple of 16500 and 990. If $\frac{16500}{a}$ and $\frac{990}{b}$ are both integers, what is the minimum value of $a + b$?

p18. Let x be a positive real number so that $x - \frac{1}{x} = 1$. Compute $x^8 - \frac{1}{x^8}$.

p19. Six people sit around a round table. Each person rolls a standard 6-sided die. If no two people sitting next to each other rolled the same number, we will say that the roll is valid. How many di erent rolls are valid?

p20. Given that $\frac{1}{31} = 0.\overline{a_1a_2a_3a_4a_5...a_n}$ (that is, $\frac{1}{31}$ can be written as the repeating decimal expansion $0.a_1a_2...a_na_1a_2...a_na_1a_2...$), what is the minimum value of n?

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Ind. Tie p1. A bus leaves San Mateo with n fairies on board. When it stops in San Francisco, each fairy gets off, but for each fairy that gets off, n fairies get on. Next it stops in Oakland where 6 times as many fairies get off as there were in San Mateo. Finally the bus arrives at Berkeley, where the remaining 391 fairies get off. How many fairies were on the bus in San Mateo?

p2. Let a and b be two real solutions to the equation $x^2 + 8x - 209 = 0$. Find $\frac{ab}{a+b}$. Express your answer as a decimal or a fraction in lowest terms.

p3. Let a, b, and c be positive integers such that the least common multiple of a and b is 25 and the least common multiple of b and c is 27. Find abc.

p4. It takes Justin 15 minutes to finish the Speed Test alone, and it takes James 30 minutes to finish the Speed Test alone. If Justin works alone on the Speed Test for 3 minutes, then how many minutes will it take Justin and James to finish the rest of the test working together? Assume each problem on the Speed Test takes the same amount of time.

p5. Angela has 128 coins. 127 of them have the same weight, but the one remaining coin is heavier than the others. Angela has a balance that she can use to compare the weight of two collections of coins against each other (that is, the balance will not tell Angela the weight of a collection of coins, but it will say which of two collections is heavier). What is the minumum number of weighings Angela must perform to guarantee she can determine which coin is heavier?

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