Junior Balkan MO 2016

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1	A trapezoid $ABCD$ ($AB\|\|CF$,$AB > CD$) is circumscribed.The incircle of the triangle ABC touches the lines AB and AC at the points M and N,respectively.Prove that the incenter of the trapezoid $ABCD$ lies on the line MN.
2	Let a,b,cbe positive real numbers.Prove that $\frac{8}{(a+b)^2+4abc}+\frac{8}{(b+c)^2+4abc}+\frac{8}{(a+c)^2+4abc}+a^2+b^2+c^2 \ge \frac{8}{a+3}+\frac{8}{b+3}+\frac{8}{c+3}$.
3	Find all triplets of integers (a,b,c) such that the number $$N = \frac{(a-b)(b-c)(c-a)}{2} + 2$$ is a power of 2016. (A power of 2016 is an integer of form 2016^n,where n is a non-negative integer.)
4	A 5×5 table is called regular f each of its cells contains one of four pairwise distinct real numbers,such that each of them occurs exactly one in every 2×2 subtable.The sum of all numbers of a regular table is called the total sum of the table.With any four numbers,one constructs all possible regular tables,computes their total sums and counts the distinct outcomes.Determine the maximum possible count.