Greece Team Selection Test 2016

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1 Given is the sequence $(a_n)_{n\geq 0}$ which is defined as follows:$a_0=3$ and $a_{n+1}-a_n=n(a_n-1)$, $\forall n\geq 0$.

Determine all positive integers m such that $\gcd (m,a_n)=1$, $\forall n\geq 0$.

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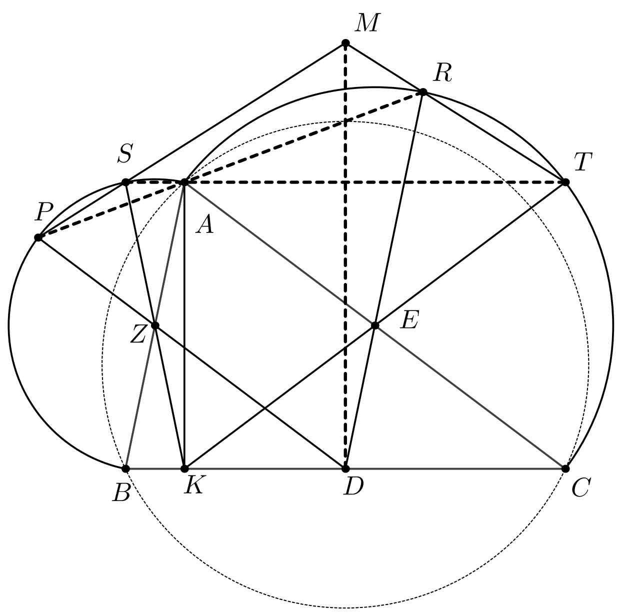
2 Given is a triangle $\triangle ABC$,with $AB<AC<BC$,inscribed in circle $c(O,R)$.Let D,E,Z be the midpoints of BC,CA,AB respectively,and K the foot of the altitude from A.At the exterior of $\triangle ABC$ and with the sides AB,AC as diameters,we construct the semicircles c_1,c_2 respectively.Suppose that $P\equiv DZ\cap c_1$, $S\equiv KZ\cap c_1$ and $R\equiv DE\cap c_2$, $T\equiv KE\cap c_2$.Finally,let M be the intersection of the lines PS,RT.

i. Prove that the lines PR,ST intersect at A.

ii. Prove that the lines $PR\cap MD$ intersect on c.



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3 Determine all functions $f:\mathbb{Z}\rightarrow \mathbb{Z}$ with the property that

$$f(x-f(y))=f(f(x))-f(y)-1$$

holds for all $x,y\in \mathbb{Z}$.

4 For a finite set A of positive integers, a partition of A into two disjoint nonempty subsets A_1 and A_2 is *good* if the least common multiple of the elements in A_1 is equal to the greatest common divisor of the elements in A_2. Determine the minimum value of n such that there exists a set of n positive integers with exactly 2015 good partitions.