Bosnia and Herzegovina Team Selection Test 2006

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–	Day 1
1	Let Z shape be a shape such that it covers (i,j), $(i,j+1)$, $(i+1,j+1)$, $(i+2,j+1)$ and $(i+2,j+2)$ where (i,j) stands for cell in i-th row and j-th column on an arbitrary table. At least how many Z shapes is necessary to cover one 8×8 table if every cell of a Z shape is either cell of a table or it is outside the table (two Z shapes can overlap and Z shapes can rotate)?
2	It is given a triangle $\triangle ABC$. Determine the locus of center of rectangle inscribed in triangle ABC such that one side of rectangle lies on side AB.
3	Prove that for every positive integer n holds inequality $\{n\sqrt{7}\} > \frac{3\sqrt{7}}{14n}$, where $\{x\}$ is fractional part of x.
–	Day 2
4	Prove that every infinite arithmetic progression a, $a+d$, $a+2d$,... where a and d are positive integers, contains infinte geometric progression b, bq, bq^2,... where b and q are also positive integers
5	Triangle ABC is inscribed in circle with center O. Let P be a point on arc AB which does not contain point C. Perpendicular from point P on line BO intersects side AB in point S, and side BC in T. Perpendicular from point P on line AO intersects side AB in point Q, and side AC in R. (i) Prove that triangle PQS is isosceles (ii) Prove that $\frac{PQ}{QR} = \frac{ST}{PQ}$
6	Let a_1, a_2,...,a_n be constant real numbers and x be variable real number x. Let $f(x) = cos(a_1 + x) + \frac{cos(a_2+x)}{2} + \frac{cos(a_3+x)}{2^2} + ... + \frac{cos(a_n+x)}{2^{n-1}}$. If $f(x_1) = f(x_2) = 0$, prove that $x_1 - x_2 = m\pi$, where m is integer.