Rice Math Tournament 2008

www.artofproblemsolving.com/community/c2976671

by parmenides51

– Team Round

– **p1**. Find the maximum value of $e^{\sin x \cos x \tan x}$.

p2. A fighter pilot finds that the average number of enemy ZIG planes she shoots down is $56z - 4z^2$, where z is the number of missiles she fires. Intending to maximize the number of planes she shoots down, she orders her gunner to "Have a nap ... then fire z missiles!" where z is an integer. What should z be?

p3. A sequence is generated as follows: if the n^{th} term is even, then the $(n+1)^{th}$ term is half the n^{th} term; otherwise it is two more than twice the n^{th} term. If the first term is 10, what is the 2008^{th} term?

p4. Find the volume of the solid formed by rotating the area under the graph of $y = \sqrt{x}$ around the x-axis, with $0 \le x \le 2$.

p5. Find the volume of a regular octahedron whose vertices are at the centers of the faces of a unit cube.

p6. What is the area of the triangle with vertices $(x, 0, 0)$, $(0, y, 0)$, and $(0, 0, z)$?

p7. Daphne is in a maze of tunnels shown below. She enters at A, and at each intersection, chooses a direction randomly (including possibly turning around). Once Daphne reaches an exit, she will not return into the tunnels. What is the probability that she will exit at A?
https://cdn.artofproblemsolving.com/attachments/c/0/0f8777e9ac9cbe302f042d040e8864d68cadb
png

p8. In triangle AXE, T is the midpoint of $\overline{EX}$, and P is the midpoint of $\overline{ET}$. If triangle APE is equilateral, find $\cos(m\angle XAE)$.

p9. In rectangle $XKCD$, J lies on $\overline{KC}$ and Z lies on $\overline{XK}$. If $\overline{XJ}$ and $\overline{KD}$ intersect at Q, $\overline{QZ} \perp \overline{XK}$, and $\frac{KC}{KJ} = n$, find $\frac{XD}{QZ}$.

p10. Bill the magician has cards A, B, and C as shown. For his act, he asks a volunteer to pick any number from 1 through 8 and tell him which cards among A, B, and C contain it. He then uses this information to guess the volunteer's number (for example, if the volunteer told Bill "A and C", he would guess "3").
One day, Bill loses card C and cannot remember which numbers were on it. He is in a hurry and randomly chooses four different numbers from 1 to 8 to write on a card. What is the probability Bill will still be able to do his trick?

A: 2 3 5 7

B: 2 4 6 7

C: 2 3 6 1

p11. Given that $f(x, y) = x^7y^8 + x^4y^{14} + A$ has root $(16, 7)$, find another root.

p12. How many nonrectangular trapezoids can be formed from the vertices of a regular octagon?

p13. If $re^{i\theta}$ is a root of $x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$, $r > 0$, and $0 \le \theta < 360$ with θ in degrees, find all possible values of θ.

p14. For what real values of n is $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\tan(x))^n dx$ defined?

p15. A parametric graph is given by

$$\begin{cases} y = \sin t \\ x = \cos t + \frac{1}{2}t \end{cases}$$

How many times does the graph intersect itself between $x = 1$ and $x = 40$?

PS. You had better use hide for answers.