Rice Math Tournament 2009

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– Team Round

1 In the future, each country in the world produces its Olympic athletes via cloning and strict training
programs. Therefore, in the finals of the 200 m free, there are two indistinguishable athletes from each
of the four countries. How many ways are there to arrange them into eight lanes?

2 Factor completely the expression $(a - b)^3 + (b - c)^3 + (c - a)^3$

3 If x and y are positive integers, and $x^4 + y^4 = 4721$, find all possible values of $x + y$

4 How many ways are there to write 657 as a sum of powers of two where each power of two is used at
most twice in the sum? For example, $256 + 256 + 128 + 16 + 1$ is a valid sum.

5 Compute $\int_0^\infty t^5 e^{-t} dt$

6 Rhombus $ABCD$ has side length 1. The size of $\angle A$ (in degrees) is randomly selected from all
real numbers between 0 and 90. Find the expected value of the area of $ABCD$.

7 An isosceles trapezoid has legs and shorter base of length 1. Find the maximum possible value
of its area

8 Simplify $\sum_{k=1}^n \frac{k^2(k-n)}{n^4}$

9 Find the shortest distance between the point $(6, 12)$ and the parabola given by the equation
$x = \frac{y^2}{2}$

10 Evaluate $\sum_{n=2009}^\infty \frac{\binom{n}{2009}}{2^n}$

11 Let z_1 and z_2 be the zeros of the polynomial $f(x) = x^2 + 6x + 11$. Compute $(1 + z_1^2 z_2)(1 + z_1 z_2^2)$.

12 A number N has 2009 positive factors. What is the maximum number of positive factors that
N^2 could have?

13   Find the remainder obtained when 17^{289} is divided by 7

14   Let a and b be integer solutions to $17a + 6b = 13$. What is the smallest possible positive value for $a - b$?

15   What is the largest integer n for which $\frac{2008!}{31^n}$ is an integer?