Math Majors of America Tournament for High Schools - Mathathon Round

www.artofproblemsolving.com/community/c2983855

by parmenides51

Sample p1. What is the largest distance between any two points on a regular hexagon with a side length of one?

p2. For how many integers $n \ge 1$ is $\frac{10^n-1}{9}$ the square of an integer?

p3. A vector in $3D$ space that in standard position in the first octant makes an angle of $\frac{\pi}{3}$ with the x axis and $\frac{\pi}{4}$ with the y axis. What angle does it make with the z axis?

p4. Compute $\sqrt{2012^2 + 2012^2 \cdot 2013^2 + 2013^2} - 2012^2$.

p5. Round $\log_2 \left(\sum_{k=0}^{32} \binom{32}{k} \cdot 3^k \cdot 5^k\right)$ to the nearest integer.

p6. Let P be a point inside a ball. Consider three mutually perpendicular planes through P. These planes intersect the ball along three disks. If the radius of the ball is 2 and $1/2$ is the distance between the center of the ball and P, compute the sum of the areas of the three disks of intersection.

p7. Find the sum of the absolute values of the real roots of the equation $x^4 - 4x - 1 = 0$.

p8. The numbers $1, 2, 3, ..., 2013$ are written on a board. A student erases three numbers a, b, c and instead writes the number

$$\frac{1}{2}(a+b+c)\left((a-b)^2 + (b-c)^2 + (c-a)^2\right).$$

She repeats this process until there is only one number left on the board. List all possible values of the remainder when the last number is divided by 3.

p9. How many ordered triples of integers (a, b, c), where $1 \le a, b, c \le 10$, are such that for every natural number n, the equation $(a+n)x^2 + (b+2n)x + c + n = 0$ has at least one real root?

Problems' source (as mentioned on official site) is Gator Mathematics Competition.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2014 Round 1

p1. A circle is inscribed inside a square such that the cube of the radius of the circle is numerically equal to the perimeter of the square. What is the area of the circle?

p2. If the coefficient of z^ky^k is 252 in the expression $(z + y)^{2k}$, find k.

p3. Let $f(x) = \frac{4x^4-2x^3-x^2-3x-2}{x^4-x^3+x^2-x-1}$ be a function defined on the real numbers where the denominator is not zero. The graph of f has a horizontal asymptote. Compute the sum of the x-coordinates of the points where the graph of f intersects this horizontal asymptote. If the graph of f does not intersect the asymptote, write 0.

Round 2

p4. How many 5-digit numbers have strictly increasing digits? For example, 23789 has strictly increasing digits, but 23889 and 23869 do not.

p5. Let
$$y =\frac{1}{1 +\frac{1}{9 + \frac{1}{5+\frac{1}{9+\frac{1}{5+...}}}}}$$
If y can be represented as $\frac{a\sqrt{b}+c}{d}$, where b is not divisible by any squares, and the greatest common divisor of a and d is 1, find the sum $a + b + c + d$.

p6. "Counting" is defined as listing positive integers, each one greater than the previous, up to (and including) an integer n. In terms of n, write the number of ways to count to n.

Round 3

p7. Suppose p, q, $2p^2 + q^2$, and $p^2 + q^2$ are all prime numbers. Find the sum of all possible values of p.

p8. Let $r(d)$ be a function that reverses the digits of the 2-digit integer d. What is the smallest 2-digit positive integer N such that for some 2-digit positive integer n and 2-digit positive integer $r(n)$, N is divisible by n and $r(n)$, but not by 11?

p9. What is the period of the function $y = (\sin(3\theta) + 6)^2 - 10(sin(3\theta) + 7) + 13$?

Round 4

p10. Three numbers a, b, c are given by $a = 2^2(\sum_{i=0}^{2} 2^i)$, $b = 2^4(\sum_{i=0}^{4} 2^i)$, and $c = 2^6(\sum_{i=0}^{6} 2^i)$. u, v, w are the sum of the divisors of a, b, c respectively, yet excluding the original number itself. What is the value of $a + b + c - u - v - w$?

p11. Compute $\sqrt{6 - \sqrt{11}} - \sqrt{6 + \sqrt{11}}$.

p12. Let $a_0, a_1, ..., a_n$ be such that $a_n \neq 0$ and

$$(1 + x + x^3)^{341}(1 + 2x + x^2 + 2x^3 + 2x^4 + x^6)^{342} = \sum_{i=0}^{n} a_i x^i.$$

Find the number of odd numbers in the sequence $a_0, a_1, ..., a_n$.

PS. You should use hide for answers. Rounds 5-7 have been posted here (`https://artofproblemsolving.com/community/c4h2781343p24424617`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2014 Round 5

p13. How many ways can we form a group with an odd number of members (plural) from 99 people? Express your answer in the form $a^b + c$, where a, b, and c are integers and a is prime.

p14. A cube is inscibed in a right circular cone such that the ratio of the height of the cone to the radius is $2 : 1$. Compute the fraction of the cone's volume that the cube occupies.

p15. Let $F_0 = 1$, $F_1 = 1$ and $F_k = F_{k-1} + F_{k-2}$. Let $P(x) = \sum_{k=0}^{99} x^{F_k}$. The remainder when $P(x)$ is divided by $x^3 - 1$ can be expressed as $ax^2 + bx + c$. Find $2a + b$.

Round 6

p16. Ankit finds a quite peculiar deck of cards in that each card has n distinct symbols on it and any two cards chosen from the deck will have exactly one symbol in common. The cards are guaranteed to not have a certain symbol which is held in common with all the cards. Ankit decides to create a function f(n) which describes the maximum possible number of cards in a set given the previous constraints. What is the value of $f(10)$?

p17. If $|x| < \frac{1}{4}$ and

$$X = \sum_{N=0}^{\infty}\sum_{n=0}^{N}\binom{N}{n}x^{2n}(2x)^{N-n}.$$

then write X in terms of x without any summation or product symbols (and without an infinite number of '$+$'s, etc.).

p18. Dietrich is playing a game where he is given three numbers a, b, c which range from $[0, 3]$ in a continuous uniform distribution. Dietrich wins the game if the maximum distance between any two numbers is no more than 1. What is the probability Dietrich wins the game?

Round 7

p19. Consider f defined by

$$f(x) = x^6 + a_1x^5 + a_2x^4 + a_3x^3 + a_4x^2 + a_5x + a_6.$$

How many tuples of positive integers $(a_1, a_2, a_3, a_4, a_5, a_6)$ exist such that $f(-1) = 12$ and $f(1) = 30$?

p20. Let a_n be the number of permutations of the numbers $S = \{1, 2, ..., n\}$ such that for all k with $1 \le k \le n$, the sum of k and the number in the kth position of the permutation is a power of 2. Compute $a_1 + a_2 + a_4 + a_8 + ... + a_{1048576}$.

p21. A 4-dimensional hypercube of edge length 1 is constructed in 4-space with its edges parallel to the coordinate axes and one vertex at the origin. Its coordinates are given by all possible permutations of $(0, 0, 0, 0)$,$(1, 0, 0, 0)$,$(1, 1, 0, 0)$,$(1, 1, 1, 0)$, and $(1, 1, 1, 1)$. The 3-dimensional hyperplane given by $x + y + z + w = 2$ intersects the hypercube at 6 of its vertices. Compute the 3-dimensional volume of the solid formed by the intersection.

PS. You should use hide for answers. Rounds 1-4 have been posted here (`https://artofproblemsolving.com/community/c4h2781335p24424563`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2015 Round 1

p1. If this mathathon has 7 rounds of 3 problems each, how many problems does it have in total? (Not a trick!)

p2. Five people, named $A, B, C, D,$ and E, are standing in line. If they randomly rearrange themselves, what's the probability that nobody is more than one spot away from where they started?

p3. At Barrios's absurdly priced fish and chip shop, one fish is worth $\$13$, one chip is worth $\$5$. What is the largest integer dollar amount of money a customer can enter with, and not be able to spend it all on fish and chips?

Round 2

p4. If there are 15 points in 4-dimensional space, what is the maximum number of hyperplanes that these points determine?

p5. Consider all possible values of $\frac{z_1-z_2}{z_2-z_3}\cdot\frac{z_1-z_4}{z_2-z_4}$ for any distinct complex numbers z_1, z_2, z_3, and z_4. How many complex numbers cannot be achieved?

p6. For each positive integer n, let $S(n)$ denote the number of positive integers $k \le n$ such that $gcd(k, n) = gcd(k + 1, n) = 1$. Find $S(2015)$.

Round 3

p7. Let P_1, P_2,..., P_{2015} be 2015 distinct points in the plane. For any $i, j \in \{1, 2, ..., 2015\}$, connect P_i and P_j with a line segment if and only if $gcd(i - j, 2015) = 1$. Define a clique to be a set of points such that any two points in the clique are connected with a line segment. Let ω be the unique positive integer such that there exists a clique with ω elements and such that there does not exist a clique with $\omega + 1$ elements. Find ω.

p8. A Chinese restaurant has many boxes of food. The manager notices that • He can divide the boxes into groups of M where M is 19, 20, or 21. • There are exactly 3 integers x less than 16 such that grouping the boxes into groups of x leaves 3 boxes left over.
Find the smallest possible number of boxes of food.

p9. If $f(x) = x|x| + 2$, then compute $\sum_{k=-1000}^{1000} f^{-1}(f(k) + f(-k) + f^{-1}(k))$.

Round 4

p10. Let ABC be a triangle with $AB = 13$, $BC = 20$, $CA = 21$. Let $ABDE$, $BCFG$, and $CAHI$

be squares built on sides AB, BC, and CA, respectively such that these squares are outside of ABC. Find the area of $DEHIFG$.

p11. What is the sum of all of the distinct prime factors of $7783 = 6^5 + 6 + 1$?

p12. Consider polyhedron $ABCDE$, where $ABCD$ is a regular tetrahedron and $BCDE$ is a regular tetrahedron. An ant starts at point A. Every time the ant moves, it walks from its current point to an adjacent point. The ant has an equal probability of moving to each adjacent point. After 6 moves, what is the probability the ant is back at point A?

PS. You should use hide for answers. Rounds 5-7 have been posted here (`https://artofproblemsolving.com/community/c4h2782011p24434676`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2015 Round 5

p13. You have a 26×26 grid of squares. Color each randomly with red, yellow, or blue. What is the expected number (to the nearest integer) of 2×2 squares that are entirely red?

p14. Four snakes are boarding a plane with four seats. Each snake has been assigned to a different seat. The first snake sits in the wrong seat. Any subsequent snake will sit in their assigned seat if vacant, if not, they will choose a random seat that is available. What is the expected number of snakes who sit in their correct seats?

p15. Let $n \ge 1$ be an integer and $a > 0$ a real number. In terms of n, find the number of solutions $(x_1, ..., x_n)$ of the equation $\sum_{i=1}^n (x_i^2 + (a - x_i)^2) = na^2$ such that x_i belongs to the interval $[0, a]$, for $i = 1, 2, ..., n$.

Round 6

p16. All roots of
$$\prod_{n=1}^{25} \prod_{k=0}^{2n} (-1)^k \cdot x^k = 0$$
are written in the form $r(\cos \phi + i \sin \phi)$ for $i^2 = -1$, $r > 0$, and $0 \le \phi < 2\pi$. What is the smallest positive value of ϕ in radians?

p17. Find the sum of the distinct real roots of the equation

$$\sqrt[3]{x^2-2x+1}+\sqrt[3]{x^2-x-6}=\sqrt[3]{2x^2-3x-5}.$$

p18. If a and b satisfy the property that $a2^n+b$ is a square for all positive integers n, find all possible value(s) of a.

Round 7

p19. Compute $(1-\cot 19^o)(1-\cot 26^o)$.

p20. Consider triangle ABC with $AB=3$, $BC=5$, and $\angle ABC=120^o$. Let point E be any point inside ABC. The minimum of the sum of the squares of the distances from E to the three sides of ABC can be written in the form a/b , where a and b are natural numbers such that the greatest common divisor of a and b is 1. Find $a+b$.

p21. Let $m\neq 1$ be a square-free number (an integer – possibly negative – such that no square divides m). We denote $Q(\sqrt{m})$ to be the set of all $a+b\sqrt{m}$ where a and b are rational numbers. Now for a fixed m, let S be the set of all numbers x in $Q(\sqrt{m})$ such that x is a solution to a polynomial of the form: $x^n+a_1x^{n-1}+....+a_n=0$, where a_0, ..., a_n are integers. For many integers m, $S=Z[\frac{m}{]}=\{a+b\sqrt{m}\}$ where a and b are integers. Give a classification of the integers for which this is not true. (Hint: It is true for $m=-1$ and 2.)

PS. You should use hide for answers. Rounds 1-4 have been posted here (https://artofproblemsolving.com/community/c4h2782002p24434611). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2016 Round 1

p1. This year, the Mathathon consists of 7 rounds, each with 3 problems. Another math test, Aspartaime, consists of 3 rounds, each with 5 problems. How many more problems are on the Mathathon than on Aspartaime?

p2. Let the solutions to $x^3+7x^2-242x-2016=0$be a,b, and c. Find $a^2+b^2+c^2$. (You might find it helpful to know that the roots are all rational.)

p3. For triangle ABC, you are given $AB=8$ and $\angle A=30^o$. You are told that BC will be chosen from amongst the integers from 1 to 10, inclusive, each with equal probability. What is the probability that once the side length BC is chosen there is exactly one possible triangle ABC?

Round 2

p4. It's raining! You want to keep your cat warm and dry, so you want to put socks, rain boots, and plastic bags on your cat's four paws. Note that for each paw, you must put the sock on before the boot, and the boot before the plastic bag. Also, the items on one paw do not affect the items you can put on another paw. How many different orders are there for you to put all twelve items of rain footwear on your cat?

p5. Let a be the square root of the least positive multiple of 2016 that is a square. Let b be the cube root of the least positive multiple of 2016 that is a cube. What is $a - b$?

p6. Hypersomnia Cookies sells cookies in boxes of $6, 9$ or 10. You can only buy cookies in whole boxes. What is the largest number of cookies you cannot exactly buy? (For example, you couldn't buy 8 cookies.)

Round 3

p7. There is a store that sells each of the 26 letters. All letters of the same type cost the same amount (i.e. any 'a' costs the same as any other 'a'), but different letters may or may not cost different amounts. For example, the cost of spelling "trade" is the same as the cost of spelling "tread," even though the cost of using a 't' may be different from the cost of an 'r.' If the letters to spell out 1 cost $\$1001$, the letters to spell out 2 cost $\$1010$, and the letters to spell out 11 cost $\$2015$, how much do the letters to spell out 12 cost?

p8. There is a square $ABCD$ with a point P inside. Given that $PA = 6$, $PB = 9$, $PC = 8$. Calculate PD.

p9. How many ordered pairs of positive integers (x, y) are solutions to $x^2 - y^2 = 2016$?

Round 4

p10. Given a triangle with side lengths $5, 6$ and 7, calculate the sum of the three heights of the triangle.

p11. There are 6 people in a room. Each person simultaneously points at a random person in the room that is not him/herself. What is the probability that each person is pointing at someone who is pointing back to them?

p12. Find all x such that $\sum_{i=0}^{\infty} ix^i = \frac{3}{4}$.

PS. You should use hide for answers. Rounds 5-7 have been posted here (https://artofproblemsolving.com/community/c4h2782837p24446063). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2016 Round 5

p13. Let $\{a\}_{n\ge 1}$ be an arithmetic sequence, with $a_1 = 0$, such that for some positive integers k and x we have $a_{k+1} = {k \choose x}$, $a_{2k+1} = {k \choose x+1}$, and $a_{3k+1} = {k \choose x+2}$. Let $\{b\}_{n\ge 1}$ be an arithmetic sequence of integers with $b_1 = 0$. Given that there is some integer m such that $b_m = {k \choose x}$, what is the number of possible values of b_2?

p14. Let $A = arcsin \left(\frac{1}{4}\right)$ and $B = arcsin \left(\frac{1}{7}\right)$. Find $\sin(A + B) \sin(A - B)$.

p15. Let $\{f_i\}_{i=1}^{9}$ be a sequence of continuous functions such that $f_i : R \to Z$ is continuous (i.e. each f_i maps from the real numbers to the integers). Also, for all i, $f_i(i) = 3^i$. Compute $\sum_{k=1}^{9} f_k \circ f_{k-1} \circ ... \circ f_1(3^{-k})$.

Round 6

p16. If x and y are integers for which $\frac{10x^3+10x^2y+xy^3+y^4}{203} = 1134341$ and $x - y = 1$, then compute $x + y$.

p17. Let T_n be the number of ways that n letters from the set $\{a, b, c, d\}$ can be arranged in a line (some letters may be repeated, and not every letter must be used) so that the letter a occurs an odd number of times. Compute the sum $T_5 + T_6$.

p18. McDonald plays a game with a standard deck of 52 cards and a collection of chips numbered 1 to 52. He picks 1 card from a fully shuffled deck and 1 chip from a bucket, and his score is the product of the numbers on card and on the chip. In order to win, McDonald must obtain a score that is a positive multiple of 6. If he wins, the game ends; if he loses, he eats a burger, replaces the card and chip, shuffles the deck, mixes the chips, and replays his turn. The probability that he wins on his third turn can be written in the form $\frac{x^2 \cdot y}{z^3}$ such that x, y, and z are relatively prime positive integers. What is $x + y + z$?

(NOTE: Use Ace as 1, Jack as 11, Queen as 12, and King as 13)

Round 7

p19. Let $f_n(x) = ln(1 + x^{2^n} + x^{2^{n+1}} + x^{3 \cdot 2^n})$. Compute $\sum_{k=0}^{\infty} f_{2k}\left(\frac{1}{2}\right)$.

p20. $ABCD$ is a quadrilateral with $AB = 183$, $BC = 300$, $CD = 55$, $DA = 244$, and $BD = 305$. Find AC.

p21. Define $\overline{xyz(t + 1)} = 1000x + 100y + 10z + t + 1$ as the decimal representation of a four digit integer. You are given that $3^x5^y7^z2^t = \overline{xyz(t + 1)}$ where x, y, z, and t are non-negative integers such that t is odd and $0 \le x, y, z,(t + 1) \le 9$. Compute$3^x5^y7^z$

PS. You should use hide for answers. Rounds 1-4 have been posted here (https://artofproblemsolving.com/community/c4h2782822p24445934). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2017 Round 1

p1. Jom and Terry both flip a fair coin. What is the probability both coins show the same side?

p2. Under the same standard air pressure, when measured in Fahrenheit, water boils at 212^o F and freezes at 32^o F. At thesame standard air pressure, when measured in Delisle, water boils at 0 D and freezes at 150 D. If x is today's temperature in Fahrenheit and y is today's temperature expressed in Delisle, we have $y = ax + b$. What is the value of $a + b$? (Ignore units.)

p3. What are the last two digits of $5^1 + 5^2 + 5^3 + \ + 5^{10} + 5^{11}$?

Round 2

p4. Compute the average of the magnitudes of the solutions to the equation $2x^4 + 6x^3 + 18x^2 + 54x + 162 = 0$.

p5. How many integers between 1 and 1000000 inclusive are both squares and cubes?

p6. Simon has a deck of 48 cards. There are 12 cards of each of the following 4 suits: fire, water, earth, and air. Simon randomly selects one card from the deck, looks at the card, returns the selected card to the deck, and shuffles the deck. He repeats the process until he selects an air card. What is the probability that the process ends without Simon selecting a fire or a water card?

Round 3

p7. Ally, Beth, and Christine are playing soccer, and Ally has the ball. Each player has a decision: to pass the ball to a teammate or to shoot it. When a player has the ball, they have a probability p of shooting, and $1-p$ of passing the ball. If they pass the ball, it will go to one of the other two teammates with equal probability. Throughout the game, p is constant. Once the ball has been shot, the game is over. What is the maximum value of p that makes Christine's total probability of shooting the ball $\frac{3}{20}$?

p8. If x and y are real numbers, then what is the minimum possible value of the expression $3x^2 - 12xy + 14y^2$ given that $x - y = 3$?

p9. Let ABC be an equilateral triangle, let D be the reflection of the incenter of triangle ABC over segment AB, and let E be the reflection of the incenter of triangle ABD over segment AD. Suppose the circumcircle Ω of triangle ADE intersects segment AB again at X. If the length of AB is 1, find the length of AX.

Round 4

p10. Elaine has c cats. If she divides c by 5, she has a remainder of 3. If she divides c by 7, she has a remainder of 5. If she divides c by 9, she has a remainder of 7. What is the minimum value c can be?

p11. Your friend Donny offers to play one of the following games with you. In the first game, he flips a fair coin and if it is heads, then you win. In the second game, he rolls a 10-sided die (its faces are numbered from 1 to 10) x times. If, within those x rolls, the number 10 appears, then you win. Assuming that you like winning, what is the highest value of x where you would prefer to play the coin-flipping game over the die-rolling game?

p12. Let be the set $X = \{0, 1, 2, ..., 100\}$. A subset of X, called N, is defined as the set that contains every element x of X such that for any positive integer n, there exists a positive integer k such that n can be expressed in the form $n = x^{a_1} + x^{a_2} + ... + x^{a_k}$, for some integers $a_1, a_2, ..., a_k$ that satisfy $0 \le a_1 \le a_2 \le ... \le a_k$. What is the sum of the elements in N?

PS. You should use hide for answers. Rounds 5-7 have be posted here (https://artofproblemsolving.com/community/c4h2782880p24446580). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2017 Round 5

p13. Points A, B, C, and D lie in a plane with $AB = 6$, $BC = 5$, and $CD = 5$, and AB is perpendicular to BC. Point E lies on line AD such that $D \neq E$, $AE = 3$ and $CE = 5$. Find DE.

p14. How many ordered pairs of integers (x, y) are solutions to $x^2y = 36 + y$?

p15. Chicken nuggets come in boxes of two sizes, a nuggets per box and b nuggets per box. We know that 899 nuggets is the largest number of nuggets we cannot obtain with some combination of a-sized boxes and b-sized boxes. How many different pairs (a, b) are there with $a < b$?

Round 6

p16. You are playing a game with coins with your friends Alice and Bob. When all three of you flip your respective coins, the majority side wins. For example, if Alice, Bob, and you flip Heads, Tails, Heads in that order, then you win. If Alice, Bob, and you flip Heads, Heads, Tails in that order, then you lose. Notice that more than one person will ”win.” Alice and Bob design their coins as follows: a value p is chosen randomly and uniformly between 0 and 1. Alice then makes a biased coin that lands on heads with probability p, and Bob makes a biased coin that lands on heads with probability $1 - p$. You design your own biased coin to maximize your chance of winning without knowing p. What is the probability that you win?

p17. There are N distinct students, numbered from 1 to N. Each student has exactly one hat: y students have yellow hats, b have blue hats, and r have red hats, where $y + b + r = N$ and $y, b, r > 0$. The students stand in a line such that all the r people with red hats stand in front of all the b people with blue hats. Anyone wearing red is standing in front of everyone wearing blue. The y people with yellow hats can stand anywhere in the line. The number of ways for the students to stand in a line is 2016. What is $100y + 10b + r$?

p18. Let P be a point in rectangle $ABCD$ such that $\angle APC = 135^o$ and $\angle BPD = 150^o$. Suppose furthermore that the distance from P to AC is 18. Find the distance from P to BD.

Round 7

p19. Let triangle ABC be an isosceles triangle with $|AB| = |AC|$. Let D and E lie on AB and AC, respectively. Suppose $|AD| = |BC| = |EC|$ and triangle ADE is isosceles. Find the sum of all possible values of $\angle BAC$ in radians. Write your answer in the form $2arcsin\left(\frac{a}{b}\right) + \frac{c}{d}\pi$, where $\frac{a}{b}$ and $\frac{c}{d}$ are in lowest terms, $-1 \le \frac{a}{b} \le 1$, and $-1 \le \frac{c}{d} \le 1$.

p20. Kevin is playing a game in which he aims to maximize his score. In the n^{th} round, for $n \ge 1$, a real number between 0 and $\frac{1}{3^n}$ is randomly generated. At each round, Kevin can either choose to have the randomly generated number from that round as his score and end the game, or he can choose to pass on the number and continue to the next round. Once Kevin passes on a number, he CANNOT claim that number as his score. Kevin may continue playing for as many rounds as he wishes. If Kevin plays optimally, the expected value of his score is $a + b\sqrt{c}$ where a, b, and c are integers and c is positive and not divisible by any positive perfect square other than 1. What is $100a + 10b + c$?

p21. Lisa the ladybug (a dimensionless ladybug) lives on the coordinate plane. She begins at the origin and walks along the grid, at each step moving either right or up one unit. The path she takes ends up at $(2016, 2017)$. Define the "area" of a path as the area below the path and above the x-axis. The sum of areas over all paths that Lisa can take can be represented as as $a \cdot \binom{4033}{2016}$. What is the remainder when a is divided by 1000?

PS. You should use hide for answers. Rounds 1-4 have been posted here (`https://artofproblemsolving.com/community/c4h2782871p24446475`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2018 Round 1

p1. Elaine creates a sequence of positive integers $\{s_n\}$. She starts with $s_1 = 2018$. For $n \ge 2$, she sets $s_n = \frac{1}{2}s_{n-1}$ if s_{n-1} is even and $s_n = s_{n-1} + 1$ if s_{n-1} is odd. Find the smallest positive integer n such that $s_n = 1$, or submit "0" as your answer if no such n exists.

p2. Alice rolls a fair six-sided die with the numbers 1 through 6, and Bob rolls a fair eight-sided die with the numbers 1 through 8. Alice wins if her number divides Bob's number, and Bob wins otherwise. What is the probability that Alice wins?

p3. Four circles each of radius $\frac{1}{4}$ are centered at the points $\left(\pm\frac{1}{4}, \pm\frac{1}{4}\right)$, and ther exists a fifth circle is externally tangent to these four circles. What is the radius of this fifth circle?

Round 2

p4. If Anna rows at a constant speed, it takes her two hours to row her boat up the river (which flows at a constant rate) to Bob's house and thirty minutes to row back home. How many minutes would it take Anna to row to Bob's house if the river were to stop flowing?

p5. Let $a_1 = 2018$, and for $n \geq 2$ define $a_n = 2018^{a_{n-1}}$. What is the ones digit of a_{2018}?

p6. We can write $(x+35)^n = \sum_{i=0}^{n} c_i x^i$ for some positive integer n and real numbers c_i. If $c_0 = c_2$, what is n?

Round 3

p7. How many positive integers are factors of $12!$ but not of $(7!)^2$?

p8. How many ordered pairs $(f(x), g(x))$ of polynomials of degree at least 1 with integer coefficients satisfy $f(x)g(x) = 50x^6 - 3200$?

p9. On a math test, Alice, Bob, and Carol are each equally likely to receive any integer score between 1 and 10 (inclusive). What is the probability that the average of their three scores is an integer?

Round 4

p10. Find the largest positive integer N such that

$$(a-b)(a-c)(a-d)(a-e)(b-c)(b-d)(b-e)(c-d)(c-e)(d-e)$$

is divisible by N for all choices of positive integers $a > b > c > d > e$.

p11. Let $ABCDE$ be a square pyramid with $ABCD$ a square and E the apex of the pyramid. Each side length of $ABCDE$ is 6. Let $ABCDD'C'B'A'$ be a cube, where AA', BB', CC', DD' are edges of the cube. Andy the ant is on the surface of $EABCDD'C'B'A'$ at the center of triangle ABE (call this point G) and wants to crawl on the surface of the cube to D'. What is the length the shortest path from G to D'? Write your answer in the form $\sqrt{a + b\sqrt{3}}$, where a and b are positive integers.

p12. A six-digit palindrome is a positive integer between $100,000$ and $999,999$ (inclusive) which is the same read forwards and backwards in base ten. How many composite six-digit palindromes are there?

PS. You should use hide for answers. Rounds 5-7 have been posted here (https://artofproblemsolving.com/community/c4h2784943p24473026). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2018 Round 5

p13. Circles ω_1, ω_2, and ω_3 have radii 8, 5, and 5, respectively, and each is externally tangent to the other two. Circle ω_4 is internally tangent to ω_1, ω_2, and ω_3, and circle ω_5 is externally tangent to the same three circles. Find the product of the radii of ω_4 and ω_5.

p14. Pythagoras has a regular pentagon with area 1. He connects each pair of non-adjacent vertices with a line segment, which divides the pentagon into ten triangular regions and one pentagonal region. He colors in all of the obtuse triangles. He then repeats this process using the smaller pentagon. If he continues this process an infinite number of times, what is the total area that he colors in? Please rationalize the denominator of your answer.

p15. Maisy arranges 61 ordinary yellow tennis balls and 3 special purple tennis balls into a $4\times4\times4$ cube. (All tennis balls are the same size.) If she chooses the tennis balls' positions in the cube randomly, what is the probability that no two purple tennis balls are touching?

Round 6

p16. Points A, B, C, and D lie on a line (in that order), and $\triangle BCE$ is isosceles with $\overline{BE} = \overline{CE}$. Furthermore, F lies on $\overline{BE}$ and G lies on $\overline{CE}$ such that $\triangle BFD$ and $\triangle CGA$ are both congruent to $\triangle BCE$. Let H be the intersection of $\overline{DF}$ and $\overline{AG}$, and let I be the intersection of $\overline{BE}$ and $\overline{AG}$. If $m\angle BCE = arcsin\left(\frac{12}{13}\right)$, what is $\frac{\overline{HI}}{\overline{FI}}$?

p17. Three states are said to form a tri-state area if each state borders the other two. What is the maximum possible number of tri-state areas in a country with fifty states? Note that states must be contiguous and that states touching only at "corners" do not count as bordering.

p18. Let a, b, c, d, and e be integers satisfying
$$2(\sqrt[3]{2})^2 + \sqrt[3]{2}a + 2b + (\sqrt[3]{2})^2c + \sqrt[3]{2}d + e = 0$$
and
$$25\sqrt{5}i + 25a - 5\sqrt{5}ib - 5c + \sqrt{5}id + e = 0$$

where $i = \sqrt{-1}$. Find $|a + b + c + d + e|$.

Round 7

p19. What is the greatest number of regions that 100 ellipses can divide the plane into? Include the unbounded region.

p20. All of the faces of the convex polyhedron P are congruent isosceles (but NOT equilateral) triangles that meet in such a way that each vertex of the polyhedron is the meeting point of either ten base angles of the faces or three vertex angles of the faces. (An isosceles triangle has two base angles and one vertex angle.) Find the sum of the numbers of faces, edges, and vertices of P.

p21. Find the number of ordered 2018-tuples of integers $(x_1, x_2, ...x_{2018})$, where each integer is between -2018^2 and 2018^2 (inclusive), satisfying

$$6(1x_1 + 2x_2 + ... + 2018x_{2018})^2 \geq (2018)(2019)(4037)(x_1^2 + x_2^2 + ... + x_{2018}^2).$$

PS. You should use hide for answers. Rounds 1-4 have been posted here (https://artofproblemsolving.com/community/c4h2784936p24472982). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2019 Round 1

p1. A small pizza costs $\$4$ and has 6 slices. A large pizza costs $\$9$ and has 14 slices. If the MMATHS organizers got at least 400 slices of pizza (having extra is okay) as cheaply as possible, how many large pizzas did they buy?

p2. Rachel flips a fair coin until she gets a tails. What is the probability that she gets an even number of heads before the tails?

p3. Find the unique positive integer n that satisfies $n! \cdot (n + 1)! = (n + 4)!$.

Round 2

p4. The Portland Malt Shoppe stocks 10 ice cream flavors and 8 mix-ins. A milkshake consists of exactly 1 flavor of ice cream and between 1 and 3 mix-ins. (Mix-ins can be repeated, the number of each mix-in matters, and the order of the mix-ins doesn't matter.) How many different milkshakes can be ordered?

p5. Find the minimum possible value of the expression $(x)^2 + (x + 3)^4 + (x + 4)^4 + (x + 7)^2$, where x is a real number.

p6. Ralph has a cylinder with height 15 and volume $\frac{960}{\pi}$. What is the longest distance (staying on the surface) between two points of the cylinder?

Round 3

p7. If there are exactly 3 pairs (x, y) satisfying $x^2 + y^2 = 8$ and $x + y = (x - y)^2 + a$, what is the value of a?

p8. If n is an integer between 4 and 1000, what is the largest possible power of 2 that n^4-13n^2+36 could be divisible by?
(Your answer should be this power of 2, not just the exponent.)

p9. Find the sum of all positive integers $n \ge 2$ for which the following statement is true: "for any arrangement of n points in three-dimensional space where the points are not all collinear, you can always find one of the points such that the $n - 1$ rays from this point through the other points are all distinct."

Round 4

p10. Donald writes the number 12121213131415 on a piece of paper. How many ways can he rearrange these fourteen digits to make another number where the digit in every place value is different from what was there before?

p11. A question on Joe's math test asked him to compute $\frac{a}{b} + \frac{3}{4}$, where a and b were both integers. Because he didn't know how to add fractions, he submitted $\frac{a+3}{b+4}$ as his answer. But it turns out that he was right for these particular values of a and b! What is the largest possible value that a could have been?

p12. Christopher has a globe with radius r inches. He puts his finger on a point on the equator. He moves his finger 5π inches North, then π inches East, then 5π inches South, then 2π inches West. If he ended where he started, what is the largest possible value of r?

PS. You should use hide for answers. Rounds 5-7 have be posted here (https://artofproblemsolving.com/community/c4h2789002p24519497). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2019 Round 5

p13. Suppose $\triangle ABC$ is an isosceles triangle with $\overline{AB} = \overline{BC}$, and X is a point in the interior of $\triangle ABC$. If $m\angle ABC = 94^o$, $m\angle ABX = 17^o$, and $m\angle BAX = 13^o$, then what is $m\angle BXC$ (in degrees)?

p14. Find the remainder when $\sum_{n=1}^{2019} 1 + 2n + 4n^2 + 8n^3$ is divided by 2019.

p15. How many ways can you assign the integers 1 through 10 to the variables $a, b, c, d, e, f, g, h, i$, and j in some order such that $a < b < c < d < e, f < g < h < i, a < g, b < h, c < i, f < b, g < c$, and $h < d$?

Round 6

p16. Call an integer n equi-powerful if n and n^2 leave the same remainder when divided by 1320. How many integers between 1 and 1320 (inclusive) are equi-powerful?

p17. There exists a unique positive integer $j \le 10$ and unique positive integers n_j , n_{j+1}, ..., n_{10} such that
$$j \le n_j < n_{j+1} < ... < n_{10}$$
and
$$\binom{n_{10}}{10} + \binom{n_9}{9} + ... + \binom{n_j}{j} = 2019.$$
Find $n_j + n_{j+1} + ... + n_{10}$.

p18. If n is a randomly chosen integer between 1 and 390 (inclusive), what is the probability that $26n$ has more positive factors than $6n$?

Round 7

p19. Suppose S is an n-element subset of $\{1, 2, 3, ..., 2019\}$. What is the largest possible value of n such that for every $2 \le k \le n$, k divides exactly $n - 1$ of the elements of S?

p20. For each positive integer n, let $f(n)$ be the fewest number of terms needed to write n as a sum of factorials. For example, $f(28) = 3$ because $4! + 2! + 2! = 28$ and 28 cannot be written as the sum of fewer than 3 factorials. Evaluate $f(1) + f(2) + ... + f(720)$.

p21. Evaluate $\sum_{n=1}^{\infty} \frac{\phi(n)}{101^n-1}$, where $\phi(n)$ is the number of positive integers less than or equal to n that are relatively prime to n.

PS. You should use hide for answers. Rounds 1-4 have been posted here (https://artofproblemsolving.com/community/c4h2788993p24519281). Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2020 Round 1

p1. Let n be a two-digit positive integer. What is the maximum possible sum of the prime factors of $n^2 - 25$?

p2. Angela has ten numbers $a_1, a_2, a_3, ..., a_{10}$. She wants them to be a permutation of the numbers $\{1, 2, 3, ..., 10\}$ such that for each $1 \le i \le 5$, $a_i \le 2i$, and for each $6 \le i \le 10$, $a_i \le -10$. How many ways can Angela choose a_1 through a_{10}?

p3. Find the number of three-by-three grids such that • the sum of the entries in each row, column, and diagonal passing through the center square is the same, and • the entries in the nine squares are the integers between 1 and 9 inclusive, each integer appearing in exactly one square.

Round 2

p4. Suppose that $P(x)$ is a quadratic polynomial such that the sum and product of its two roots are equal to each other. There is a real number a that $P(1)$ can never be equal to. Find a.

p5. Find the number of ordered pairs (x, y) of positive integers such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{k}$ and k is a factor of 60.

p6. Let ABC be a triangle with $AB = 5$, $AC = 4$, and $BC = 3$. With $B = B_0$ and $C = C_0$, define the infinite sequences of points $\{B_i\}$ and $\{C_i\}$ as follows: for all $i \ge 1$, let B_i be the foot of the perpendicular from C_{i-1} to AB, and let C_i be the foot of the perpendicular from B_i to AC. Find $C_0C_1(AC_0 + AC_1 + AC_2 + AC_3 + ...)$.

Round 3

p7. If $\ell_1, \ell_2, ..., \ell_{10}$ are distinct lines in the plane and $p_1, ..., p_{100}$ are distinct points in the plane, then what is the maximum possible number of ordered pairs (ℓ_i, p_j) such that p_j lies on ℓ_i?

p8. Before Andres goes to school each day, he has to put on a shirt, a jacket, pants, socks, and shoes. He can put these clothes on in any order obeying the following restrictions: socks come before shoes, and the shirt comes before the jacket. How many distinct orders are there for Andres to put his clothes on?

p9. There are ten towns, numbered 1 through 10, and each pair of towns is connected by a road. Define a backwards move to be taking a road from some town a to another town b such that $a > b$, and define a forwards move to be taking a road from some town a to another town b such that $a < b$. How many distinct paths can Ali take from town 1 to town 10 under the conditions that • she takes exactly one backwards move and the rest of her moves are forward moves, and • the only time she visits town 10 is at the very end?
One possible path is $1 \to 3 \to 8 \to 6 \to 7 \to 8 \to 10$.

Round 4

p10. How many prime numbers p less than 100 have the properties that $p^5 - 1$ is divisible by 6 and $p^6 - 1$ is divisible by 5?

p11. Call a four-digit integer $\overline{d_1d_2d_3d_4}$ *primed* if
1) d_1, d_2, d_3, and d_4 are all prime numbers, and
2) the two-digit numbers $\overline{d_1d_2}$ and $\overline{d_3d_4}$ are both prime numbers.
Find the sum of all primed integers.

p12. Suppose that ABC is an isosceles triangle with $AB = AC$, and suppose that D and E lie on $\overline{AB}$ and $\overline{AC}$, respectively, with $\overline{DE} \parallel \overline{BC}$. Let r be the length of the inradius of triangle ADE. Suppose that it is possible to construct two circles of radius r, each tangent to one another and internally tangent to three sides of the trapezoid $BDEC$. If $\frac{BC}{r} = a + \sqrt{b}$ forpositive integers a and b with b squarefree, then find $a + b$.

PS. You should use hide for answers. Rounds 5-7 have been posted here (`https://artofproblemsolving.com/community/c4h2800986p24675177`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2020 Round 5

p13. A palindrome is a number that reads the same forward as backwards; for example, 121 and 36463 are palindromes. Suppose that N is the maximal possible difference between two consecutive three-digit palindromes. Find the number of pairs of consecutive palindromes (A, B) satisfying $A < B$ and $B - A = N$.

p14. Suppose that x, y, and z are complex numbers satisfying $x + \frac{1}{yz} = 5$, $y + \frac{1}{zx} = 8$, and $z + \frac{1}{xy} = 6$. Find the sum of all possible values of xyz.

p15. Let Ω be a circle with radius $25\sqrt{2}$ centered at O, and let C and J be points on Ω such that the circle with diameter $\overline{CJ}$ passes through O. Let Q be a point on the circle with diameter $\overline{CJ}$ satisfying $OQ = 5\sqrt{2}$. If the area of the region bounded by $\overline{CQ}$, $\overline{QJ}$, and minor arc JC on Ω can be expressed as $\frac{a\pi - b}{c}$ for integers a, b, and c with $gcd\ (a, c) = 1$, then find $a + b + c$.

Round 6

p16. Veronica writes N integers between 2 and 2020 (inclusive) on a blackboard, and she notices that no number on the board is an integer power of another number on the board. What is the largest possible value of N?

p17. Let ABC be a triangle with $AB = 12$, $AC = 16$, and $BC = 20$. Let D be a point on AC, and suppose that I and J are the incenters of triangles ABD and CBD, respectively. Suppose that $DI = DJ$. Find IJ^2.

p18. For each positive integer a, let $P_a = \{2a, 3a, 5a, 7a, ...\}$ be the set of all prime multiples of a. Let $f(m, n) = 1$ if P_m and P_n have elements in common, and let $f(m, n) = 0$ if P_m and P_n have no elements in common. Compute

$$\sum_{1 \le i < j \le 50} f(i, j)$$

(i.e. compute $f(1, 2) + f(1, 3)+, , , +f(1, 50) + f(2, 3) + f(2, 4)+, , , +f(49, 50)$.)

Round 7

p19. How many ways are there to put the six letters in "$MMATHS$" in a two-by-three grid such that the two "M"s do not occupy adjacent squares and such that the letter "A" is not directly above the letter "T" in the grid? (Squares are said to be adjacent if they share a side.)

p20. Luke is shooting basketballs into a hoop. He makes any given shot with fixed probability p with $p < 1$, and he shoots n shots in total with $n \ge 2$. Miraculously, in n shots, the probability that Luke makes exactly two shots in is twice the probability that Luke makes exactly one shot in! If p can be expressed as $\frac{k}{100}$ for some integer k (not necessarily in lowest terms), find the sum of all possible values for k.

p21. Let $ABCD$ be a rectangle with $AB = 24$ and $BC = 72$. Call a point P *goofy* if it satisfies the following conditions: • P lies within $ABCD$, • for some points F and G lying on sides BC and DA such that the circles with diameter BF and DG are tangent to one another, P lies on their common internal tangent.
Find the smallest possible area of a polygon that contains every single goofy point inside it.

PS. You should use hide for answers. Rounds 1-4 have been posted here (`https://artofproblemsolving.com/community/c4h2800971p24674988`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2021 Round 1

p1. Ben the bear has an algorithm he runs on positive integers- each second, if the integer is even, he divides it by 2, and if the integer is odd, he adds 1. The algorithm terminates after he reaches 1. What is the least positive integer n such that Ben's algorithm performed on n will terminate after seven seconds? (For example, if Ben performed his algorithm on 3, the algorithm would terminate after 3 seconds: $3 \to 4 \to 2 \to 1$.)

p2. Suppose that a rectangle R has length p and width q, for prime integers p and q. Rectangle S has length $p + 1$ and width $q + 1$. The absolute difference in area between S and R is 21. Find the sum of all possible values of p.

p3. Owen the origamian takes a rectangular 12×16 sheet of paper and folds it in half, along the diagonal, to form a shape. Find the area of this shape.

Round 2

p4. How many subsets of the set $\{G, O, Y, A, L, E\}$ contain the same number of consonants as vowels? (Assume that Y is a consonant and not a vowel.)

p5. Suppose that trapezoid $ABCD$ satisfies $AB = BC = 5$, $CD = 12$, and $\angle ABC = \angle BCD = 90^o$. Let AC and BD intersect at E. The area of triangle BEC can be expressed as $\frac{a}{b}$, for positive integers a and b with $gcd(a, b) = 1$. Find $a + b$.

p6. Find the largest integer n for which $\frac{101^n+103^n}{101^{n-1}+103^{n-1}}$ is an integer.

Round 3

p7. For each positive integer n between 1 and 1000 (inclusive), Ben writes down a list of n's factors, and then computes the median of that list. He notices that for some n, that median is actually a factor of n. Find the largest n for which this is true.

p8. (voided) Suppose triangle ABC has $AB = 9$, $BC = 10$, and $CA = 17$. Let x be the maximal possible area of a rectangle inscribed in ABC, such that two of its vertices lie on one side and the other two vertices lie on the other two sides, respectively. There exist three rectangles R_1, R_2, and R_3 such that each has an area of x. Find the area of the smallest region containing the set of points that lie in at least two of the rectangles R_1, R_2, and R_3.

p9. Let $a, b,$ and c be the three smallest distinct positive values of θ satisfying

$$\cos\theta + \cos 3\theta + ... + \cos 2021\theta = \sin\theta + \sin 3\theta + ... + \sin 2021\theta.$$

What is $\frac{4044}{\pi}(a + b + c)$?

Problem 8 is voided.

PS. You should use hide for answers.Rounds 4-5 have been posted here (`https://artofproblemsolving.com/community/c4h3131422p28368457`) and 6-7 here (`https://artofproblemsolving.com/community/c4h3131434p28368604`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24`

2021 Round 4

p10. How many divisors of 10^{11} have at least half as many divisors that 10^{11} has?

p11. Let $f(x, y) = \frac{x}{y} + \frac{y}{x}$ and $g(x, y) = \frac{x}{y} - \frac{y}{x}$. Then, if $\underbrace{f(f(...f(f(}_{2021fs}f(f(1, 2), g(2, 1)), 2), 2)..., 2), 2)$ can be expressed in the form $a + \frac{b}{c}$, where a, b,c are nonnegative integers such that $b < c$ and $gcd(b, c) = 1$, find $a + b + \lceil(\log_2(\log_2 c)\rceil$

p12. Let ABC be an equilateral triangle, and letDEF be an equilateral triangle such that D, E, and F lie on AB, BC, and CA, respectively. Suppose that AD and BD are positive integers, and that $\frac{[DEF]}{[ABC]} = \frac{97}{196}$. The circumcircle of triangle DEF meets AB, BC, and CA again at G, H, and I, respectively. Find the side length of an equilateral triangle that has the same area as the hexagon with vertices D, E, F, G, H, and I.

Round 5

p13. Point X is on line segment AB such that $AX = \frac{2}{5}$ and $XB = \frac{5}{2}$. Circle Ω has diameter AB and circle ω has diameter XB. A ray perpendicular to AB begins at X and intersects Ω at a point Y. Let Z be a point on ω such that $\angle YZX = 90^o$. If the area of triangle XYZ can be expressed as $\frac{a}{b}$ for positive integers a, b with $gcd(a, b) = 1$, find $a + b$.

p14. Andrew, Ben, and Clayton are discussing four different songs; for each song, each person either likes or dislikes that song, and each person likes at least one song and dislikes at least one song. As it turns out, Andrew and Ben don't like any of the same songs, but Clayton likes at least one song that Andrew likes and at least one song that Ben likes! How many possible ways could this have happened?

p15. Let triangle ABC with circumcircle Ω satisfy $AB = 39$, $BC = 40$, and $CA = 25$. Let P be a point on arc BC not containing A, and let Q and R be the reflections of P in AB and AC, respectively. Let AQ and AR meet Ω again at S and T, respectively. Given that the reflection of QR over BC is tangent to Ω , ST can be expressed as $\frac{a}{b}$ for positive integers a, b with $gcd(a, b) = 1$. Find $a + b$.

PS. You should use hide for answers. Rounds 1-3 have been posted here (`https://artofproblemsolving.com/community/c4h3131401p28368159`) and 6-7 here (`https://artofproblemsolving.com/community/c4h3131434p28368604`),Collected here (`https://artofproblemsolving.com/community/c5h2760506p241`

2021 Round 6

p16. Let ABC be a triangle with $AB = 3$, $BC = 4$, and $CA = 5$. There exist two possible points X on CA such that if Y and Z are the feet of the perpendiculars from X to AB and BC, respectively, then the area of triangle XYZ is 1. If the distance between those two possible points can be expressed as $\frac{a\sqrt{b}}{c}$ for positive integers a, b, and c with b squarefree and $gcd(a, c) = 1$, then find $a + b + c$.

p17. Let $f(n)$ be the number of orderings of $1, 2, ..., n$ such that each number is as most twice the number preceding it. Find the number of integers k between 1 and 50, inclusive, such that $f(k)$ is a perfect square.

p18. Suppose that f is a function on the positive integers such that $f(p) = p$ for any prime p, and that $f(xy) = f(x) + f(y)$ for any positive integers x and y. Define $g(n) = \sum_{k|n} f(k)$; that is, $g(n)$ is the sum of all $f(k)$ such that k is a factor of n. For example, $g(6) = f(1) + 1(2) + f(3) + f(6)$. Find the sum of all composite n between 50 and 100, inclusive, such that $g(n) = n$.

<u>Round 7</u>

p19. AJ is standing in the center of an equilateral triangle with vertices labelled A, B, and C. They begin by moving to one of the vertices and recording its label; afterwards, each minute, they move to a different vertex and record its label. Suppose that they record 21 labels in total, including the initial one. Find the number of distinct possible ordered triples (a, b, c), where a is the number of A's they recorded, b is the number of B's they recorded, and c is the number of C's they recorded.

p20. Let $S = \sum_{n=1}^{\infty}(1 - \{(2 + \sqrt{3})^n\})$, where $\{x\} = x - \lfloor x \rfloor$, the fractional part of x. If $S = \frac{\sqrt{a}-b}{c}$ for positive integers a, b, c with a squarefree, find $a + b + c$.

p21. Misaka likes coloring. For each square of a 1×8 grid, she flips a fair coin and colors in the square if it lands on heads. Afterwards, Misaka places as many 1×2 dominos on the grid as possible such that both parts of each domino lie on uncolored squares and no dominos overlap. Given that the expected number of dominos that she places can be written as $\frac{a}{b}$, for positive integers a and b with $gcd(a, b) = 1$, find $a + b$.

PS. You should use hide for answers. Rounds 1-3 have been posted here (`https://artofproblemsolving.com/community/c4h3131401p28368159`) and 4-5 here (`https://artofproblemsolving.com/community/c4h3131422p28368457`). Collected here (`https://artofproblemsolving.com/community/c5h2760506p24`