Second Round - Poland 2022

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– Day 1

1 Find all real quadruples (a,b,c,d) satisfying the system of equations

$$\begin{cases} ab+cd=6 \\ ac+bd=3 \\ ad+bc=2 \\ a+b+c+d=6. \end{cases}$$

2 Given a cyclic quadriteral $ABCD$. The circumcenter lies in the quadriteral $ABCD$. Diagonals AC and BD intersects at S. Points P and Q are the midpoints of AD and BC. Let p be a line perpendicular to AC through P, q perpendicular line to BD through Q and s perpendicular to CD through S. Prove that p, q, s intersects at one point.

3 Positive integers a, b, c satisfying the equation

$$a^3 + 4b + c = abc,$$

where $a \geq c$ and the number $p = a^2 + 2a + 2$ is a prime. Prove that p divides $a + 2b + 2$.

– Day 2

4 Given quadrilateral $ABCD$ inscribed into a circle with diagonal AC as diameter. Let E be a point on segment BC s.t. $\sphericalangle DAC = \sphericalangle EAB$. Point M is midpoint of CE. Prove that $BM = DM$.

5 Let n be an positive integer. We call n *good* when there exists positive integer k s.t. $n = k(k+1)$. Does there exist 2022 pairwise distinct *good* numbers s.t. their sum is also *good* number?

6 n players took part in badminton tournament, where n is positive and odd integer. Each two players played two matches with each other. There were no draws. Each player has won as many matches as he has lost. Prove that you can cancel half of the matches s.t. each player still has won as many matches as he has lost.