**Czech And Slovak Mathematical Olympiad, Round III, Category A, 1953**

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by byk7

**1** Find the locus of all numbers $z \in \mathbb{C}$ in complex plane satisfying

$$z + \bar{z} = a \cdot |z|,$$

where $a \in \mathbb{R}$ is given.

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**2** Let $\alpha, \beta, \gamma$ be angles of a triangle. Two of them can be expressed using an auxiliary angle $\varphi$ in a way that

$$\alpha = \varphi + \frac{\pi}{4}, \quad \beta = \pi - 3\varphi.$$

Show that $\alpha > \gamma$.

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**3** Prove that the inequality

$$(a_1 + \cdots + a_n)\left(\frac{1}{a_1} + \cdots + \frac{1}{a_n}\right) \geq n^2$$

holds for any positive numbers $a_1, \ldots, a_n$ and determine when equality occurs.

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**4** Consider skew lines $a, b$ and a plane $\rho$ that intersect both of the lines (but does not contain any of them). Choose such points $X \in a, Y \in b$ that $XY \parallel \rho$. Find the locus of midpoints $M$ of all segments $XY$, when $X$ moves along line $a$.

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