JJMO Final Round 2022

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by YII.I.

1 Find all pair of primes (p, q), such that $p^3 + 3q^3 - 32$ is also a prime.

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2 Suppose $n \geq 3$ is an integer. There are n grids on a circle. We put a stone in each grid. Find all positive integer n, such that we can perform the following operation $n-2$ times, and then there exists a grid with $n-1$ stones in it:

• Pick a grid A with at least one stone in it. And pick a positive integer $k \leq n-1$. Take all stones in the k-th grid after A in anticlockwise direction. And put then in the k-th grid after A in clockwise direction.

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3 Suppose a, b, c, x, y, z are pairwisely different real numbers. How many terms in the following can be 1 at most:

$$ax + by + cz, \qquad ax + bz + cy, \qquad ay + bx + cz,$$
$$ay + bz + cx, \qquad az + bx + cy, \qquad az + by + cx?$$

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4 In an acute triangle ABC, $AB < AC$. The perpendicular bisector of the segment BC intersects the lines AB, AC at the points D, E respectively. Denote the mid-point of DE as M. Suppose the circumcircle of $\triangle ABC$ intersects the line AM at points P and A, and M, A, P are arranged in order on the line. Prove that $\angle BPE = 90^\circ$.

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5 Find all positive integer n, such that

$$\left[\frac{n}{2^0}\right]\left[\frac{n}{2^1}\right]\ldots\left[\frac{n}{2^k}\right] + 2 \cdot 4^{[\frac{k}{2}]}$$

is a square, where k is the non-negative integer satisfying $2^k \leq n < 2^{k+1}$.
