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1 Fill in one of the three letters A, B and C in each grid of a 2×2 table. How many ways are there to fill in, such that the letters in any two grids with a common side are different?

Remark: It is legal that some character may not be written even once in the square. If two writing methods can be the same only after being rotated or flipped, we still treat them as two different methods.

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2 Find the sum of pq's for all pairs of prime numbers $p \le q$ such that $pq \mid 15(p-1)(q-1)$.

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3 A pentagon $ABCDE$ is inscribed in a circle Γ. And the quadrilateral $BCDE$ is a rectangle satisfying $BC = DE = 1$. Suppose $AB = EA = 6$. How long is the diameter of Γ?

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4 In the triangle ABC, we have $AB = 5$, $BC = 7$, $CA = 6$. Take three points D, E, F inside the edges BC, CA, AB respectively, such that $ABDE$ and $BCEF$ are both circumscribed, and the circumcircle of $\triangle BDF$ is tangent to the line EF. Find the length of the segment AE.

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5 In a 45×45 table, we choose 2022 grids randomly and paint them black. If no matter how the grids are chosen, we can find a $n \times n$ subtable such that all grids of it are black, please find the maximum value of the positive integer n.

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6 As shown in the picture, the triangles ABC, ADE, EFG are all isosceles right triangle, such that $\angle ACB = \angle AED = \angle EGF = 90^\circ$. Suppose the area of the pentagon $ABDGE$ is 23, and $AB = 8$, $FD > DG$. Find the length of the segment FD.

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7 How many arrays of non-negative integers (a, b, c, d, e) satisfy $a+b+c+d+e = 2022$, and these five numbers are all not three-digit numbers.

Remark: 0 is a one-digit number.

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8 Use 20 unit cubic to form the following solid. For each unit cubic, we assign it with an integer between 1 and 8 (including 1 and 8). Suppose for any surface of the figure, the eight assigned valuations of the cubic in this surface are different in pairs. How many ways to assign numbers that satisfy the condition.

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9 How many positive integers $n \le 2022$ satisfy the following conditions? • There is a multiple of n such that only one digit of n is 0, and all other digits are 2.

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10 For integers a, b, c, d satisfy $0 < a < b < 103$ and $0 < c < d < 103$. Two men A and B play a game. Place 103 positions on the circle. One of them are named S, and the next one in the anticlockwise direction of S is named G. At the first, a stone is placed at S. Then two players take turns to perform the following operations:

• Operation of A: Move the stone a steps or b steps clockwisely. • Operation of B: Move the stone c steps or d steps clockwisely.

A performs first. The goal of B is to put the stone at G. If B has a strategy to achieve his goal no matter how A does, then we call (a, b, c, d) a *good* array. How many good arrays are there in total?

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11 In an isosceles triangle ABC, $AB = AC$. Take a point P inside the $\triangle ABC$ to satisfy $\angle PAB = \angle PBC = \angle PCA$. If the area of $\triangle PAB$ and $\triangle PCA$ are $5, 4$ respectively. Find the length of the segment BC.

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12 A palace has 32 rooms and 40 corridors. As shown in the figure below, each room is represented by a dot, and each corridor is represented by a segment connecting the rooms. Put n robots in these rooms, such that there is at most one robot in each room. Each robot is assigned to a corridor connected to its room. And the following condition is satisfied:

• Let all robots move along the assigned corridors at the same time. They will arrive at the rooms at another ends of the corridors at the same time. During this process, any two robots will not meet each other. And each robot will arrive at a different room in the end.

Assume that the maximum value of the positive integer n we can take as above is N. How many ways to put N robots in the palace and assign the corridors to them satisfying the above condition?

Remark: Any two robot are not distinguished from each other.
