Czech And Slovak Mathematical Olympiad, Round III, Category A 1954

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by byk7

1 Solve the equation

$$ax^2 + 2(a-1)x + a - 5 = 0$$

in real numbers with respect to (real) parametr a.

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2 Let a, b complex numbers. Show that if the roots of the equation $z^2 + az + b = 0$ and 0 form a triangle with the right angle at the origin, then $a^2 = 2b \neq 0$. Also determine whether the opposite implication holds.

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3 Show that

$$\log_2 \pi + \log_4 \pi < \frac{5}{2}.$$

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4 Consider a cube $ABCDA'B'C'D$ (with $AB \perp AA' \parallel BB' \parallel CC' \parallel DD$). Let X be an inner point of the segment AB and denote Y the intersection of the edge AD and the plane $B'D'X$.
(a) Let $M = B'Y \cap D'X$. Find the locus of all Ms.
(b) Determine whether there is a quadrilateral $B'D'YX$ such that $B'M = \alpha \cdot MY, D'X = \beta \cdot MX$ for $\alpha, \beta \in \{1/2, 2\}$.
