BdMO 2020 Higher Secondary National Round problems

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Problem 1 Lazim rolls two 24-sided dice. From the two rolls, Lazim selects the die with the highest number. N is an integer not greater than 24. What is the largest possible value for N such that there is a more than 50

Problem 2 How many integers n are there subject to the constraint that $1 \leq n \leq 2020$ and n^n is a perfect square?

Problem 3 Let R be the set of all rectangles centered at the origin and with perimeter 1 (the center of a rectangle is the intersection point of its two diagonals). Let S be a region that contains all of the rectangles in R (region A contains region B, if B is completely inside of A). The minimum possible area of S has the form πa, where a is a real number. Find $1/a$.

Problem 4 56 lines are drawn on a plane such that no three of them are concurrent. If the lines intersect at exactly 594 points, what is the maximum number of them that could have the same slope?

Problem 5 In triangle ABC, $AB = 52$, $BC = 34$ and $CA = 50$. We split BC into n equal segments by placing $n - 1$ new points. Among these points are the feet of the altitude, median and angle bisector from A. What is the smallest possible value of n?

Problem 6 f is a one-to-one function from the set of positive integers to itself such that

$$f(xy) = f(x)f(y)$$

Find the minimum possible value of $f(2020)$.

Problem 7 f is a function on the set of complex numbers such that $f(z) = 1/(z*)$, where $z*$ is the complex conjugate of z. S is the set of complex numbers z such that the real part of $f(z)$ lies between $1/2020$ and $1/2018$. If S is treated as a subset of the complex plane, the area of S can be expressed as $m\pi$ where m is an integer. What is the value of m?

Problem 8 We call a permutation of the numbers 1, 2, 3, $\ldots$, n 'kawaii' if there is exactly one number that is greater than its position. For example: 1, 4, 3, 2 is a kawaii permutation (when $n = 4$) because only the number 4 is greater than its position 2. How many kawaii permutations are there if $n = 14$?

Problem 9 Bristy wants to build a special set A. She starts with $A = \{0, 42\}$. At any step, she can add an integer x to the set A if it is a root of a polynomial which uses the already existing integers in A

as coefficients. She keeps doing this, adding more and more numbers to A. After she eventually runs out of numbers to add to A, how many numbers will be in A?

Problem 10 Let $ABCD$ be a convex quadrilateral. O is the intersection of AC and BD. $AO = 3$,$BO = 4$, $CO = 5$, $DO = 6$. X and Y are points in segment AB and CD respectively, such that X, O, Y are collinear. The minimum of $\frac{XB}{XA} + \frac{YC}{YD}$ can be written as $\frac{a\sqrt{c}}{b}$, where $\frac{a}{b}$ is in lowest term and c is not divisible by any square number greater then 1. What is the value of $10a + b + c$?
