Manhattan Mathematical Olympiad 1997

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– Grade 5

– **p1.** You have bought a box which contains six unsharpened pencils. Is it possible to arrange them so that every pair of pencils touch.

p2. Is it possible to put 54 rabbits in 10 cages so that every pair of cages have a different number of rabbits, and each cage contains at least one rabbit?

p3. The year 1983 had 53 Saturdays. What day of the week was January 1, 1983?

p4. Suppose you play the following game with a friend: Your friend picks a whole number between 1 and 100. You choose a number and ask your friend to compare it with his number. He tells whether your number is bigger, smaller , or equal to his. Tell how you can always guess his number in no more than seven tries.

PS. You should use hide for answers.

– Grade 6

– **p1.** a) Is it possible to put eight line segments in the plane in such a way that each of them intersects exactly three others?
b) Is it possible with seven lines segments?

p2. Is it possible to find a whole number the product of whose digits is 66?

p3. Two clever cats have stolen a string of six sausages and are ready to eat them. In his turn, each cat breaks the string between two sausages; he then eats every sausage that is no longer attached to any other sausage. How many sausages will each cat eat?

p4. Find a digit to substitute each letter so that the correct sum written in words-

$$\begin{array}{ccccc} F & O & R & T & Y \\ & & T & E & N \\ & + & T & E & N \\ \hline S & I & X & T & Y \end{array}$$

PS. You should use hide for answers.

– Grade 7

– **p1.** Every point of the plane is painted either red or green, and there is at least one red point and at least one green point. Prove that
a) There exist two points painted the same color as each other and lying exactly 1 cm apart.
b) There exist two points painted different colors from each other, and lying exactly 1 cm apart.

p2. Two people play the following game: there are two bunches of matches. The first bunch contains m matches, the second bunch contains n matches, m and n are not zero. On his turn each player takes as many matches from only one bunch as he wants, but he must take at least one match. The loser is the player who takes the last match. Prove that if m is not equal to n, then the first player can win regardless of the strategy of the second player.

p3. Suppose you are given some numbers whose sum is 1. Is it possible for the sum of their squares to be less than one tenth?

p4. Suppose p is a prime number. Show that the number $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{p}$ is not an integer.

PS. You should use hide for answers.

– Grade 8

– **p1.** Is it possible to connect 1997 phones in a network in such a way that each phone is connected to exactly 1995 of the others?

p2. A triangle lies entirely inside of a rectangle. Prove that the perimeter of the triangle is smaller than the perimeter of the rectangle.

p3. Prove that the product of the digits of a positive integer is always less than or equal to the number itself.

p4. Consider a positive integer which, when written in a usual way, uses only the digits 0 and 1. Suppose that exactly 300 1's are used, and the rest of the digits are 0's. Can this integer be a square of another integer?

PS. You should use hide for answers.