Manhattan Mathematical Olympiad 1998

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by parmenides51

– Grades 5-6

– **p1.** Suppose we want to place the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$, in the circles in the following figure in such a way that the sums of the four numbers on each side of the triangle is the same. If we denote this sum by S, find the biggest and smallest possible value of S, for which such an arrangement if possible.
`https://cdn.artofproblemsolving.com/attachments/9/4/888c8f3498a723553aeeff1e0a66a609213c9`
`gif`

p2. One has 12 matches, each being 1 inch long. Is it possible to arrange them to form a polygon with area equal to 4 in^2?

p3. Prove that, when we divide any prime number by 30, we get a remainder which is equal to either 1 or a prime number.

p4. Is it possible to cut an arbitrary triangle into several pieces in such a way that, if we put these pieces together in a different way, we get a rectangle?

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Find all prime numbers p for which $p + 10$ and $p + 14$ are also prime.

p2. Prove that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{1999} - \frac{1}{2000} = \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + ... + \frac{1}{2000}$.
Can you generalize this formula?

p3. Suppose somebody has numbered the squares of chessboard by writing in each square one number from 1 to 64, without repeating any number. Show that there exist at least two neighboring squares such that the difference between their numbers is at least 5. (Note: Two neighboring squares are two squares which have a common side.)

p4. Suppose we have 101 points inside a square with one inch sides, placed in such a way that no three points lie on a straight line. Prove there exist 3 points such that the triangle they form has area not bigger than $1/100$ in^2.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

– Grades 9-12

– **p1.** Prove that if a prime number m has the property that m^2+2 is also prime, then m^3+2 must also be prime.

p2. Suppose n is a positive integer. Find a formula to the sum:

$$\frac{1}{1\times 2\times 3\times 4}+\frac{1}{2\times 3\times 4\times 5}+\frac{1}{3\times 4\times 5\times 6}+...+\frac{1}{n(n+1)(n+2)(n+3)}.$$

p3. John is 3 years old and he knows how to write only the digit 1. Prove that, using only the digit 1, John can write a multiple of 1999. Can you characterize all integer numbers n for which, using only the digit1, one can write a multiple of n?

p4. Suppose $100,000$ straight lines are drwan in the plane, such that any two of them intersect in one point. Suppose also that, whenever P is a common point of two lines, there always exist at least one more line passing through P. Prove that all 100.000 lines have a common point.

PS. You should use hide for answers.