Manhattan Mathematical Olympiad 2006

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– Grades 5-6

– **p1.** Is it possible to place six points in the plane and connect them by nonintersecting segments so that each point will be connected with exactly
a) Three other points?
b) Four other points?

p2. Martian bank notes can have denomination of $1, 3, 5, 25$ marts. Is it possible to change a note of 25 marts to exactly 10 notes of smaller denomination?

p3. What is bigger: $99 \cdot 99 \cdot .... \cdot 99$ (product of 20 factors) or $9999 \cdot 9999 \cdot ... \cdot 9999$ (product of 10 factors)?

p4. How many whole numbers, from 1 to 2006, are divisible neither by 5 nor by 7?

PS. You should use hide for answers.

– Grades 7-8

– **p1.** You have 10 bags of 2006 coins each and a scale which can show exact weight. You know that nine bags contain honest coins which weigh 10 grams each. The remaining bag contains false coins which weigh 11 grams each. All bags look alike, so you don't know which bag contains false coins. How to determine the bag with false coins by one weighing? (This means that you can put some coins on the scale, read the result and make a decision based on the result).

p2. Is it true that if the lengths of all three altitudes of a triangle are less than 1 centimeter, then its area is less than 100 square centimeters?

p3. Find the smallest possible whole number with the following property: its half is a square of some whole number, its third is a cube of some whole number and its fifth is the fifth power of some whole numer.

p4. Martian bus tickets have six-digit numbers, so that all tickets are numbered from 000001 to 999999. Martians think that the ticket is lucky if the sum of the first three digits is equal to the sum of the last three digits. Prove that the total sum of all 6-digit numbers which appear on the lucky tickets is divisible by 13.

PS. You should use hide for answers.

– Grades 9-12

– **p1.** Let $p_1, p_2, ..., p_{2006}$ be 2006 different whole numbers, all greater than 1. Prove that

$$\left(1-\frac{1}{p_1^2}\right)\cdot\left(1-\frac{1}{p_2^2}\right)...\left(1-\frac{1}{p_{2006}^2}\right)>\frac{1}{2}$$

p2. The billiard has the form of an $m \times n$ rectanglular table, where m and n are whole numbers. One plays a ball from the center of the table, which hits a wall of the billiard in such a way that the angle between its trajectory and the wall is 30 degrees. Prove that the ball will never hit the corner of the billiard. (Comments: You may assume that the ball is a point. The ball reflects from a wall at the same angle it hits the wall).

p3. Let $a_1, a_2, a_3, ...$ be a sequence of whole numbers (maybe with repetitions) such that for each $k \ge 1$ there are exactly k terms of the sequence which divide k. Prove that there exists a term ak in this sequence, which is equal to 1024.

p4. Prove that there are no positive integer numbers x, y, z, k such that $x^k + y^k = z^k$, and $x < k$, $y < k$.

PS. You should use hide for answers.