Manhattan Mathematical Olympiad 2007

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– Grades 5-6

– **p1.** There are 64 cities in the country Moonland. Prove that there will be at least three of them which will have the same number of rainy days in September 2007.

p2. Matches from a box are placed on the table in such a way that they form a (wrong) equality in Roman numbers (each segment on the picture below is a single match). Change a position of exactly one match (without removing or breaking it) and get a correct equality in Roman numbers:
`https://cdn.artofproblemsolving.com/attachments/8/7/447706f680e1e929144e7161ca097f9538e83`
`png`

p3. A craftsman has 4 oz of paint in order to paint all faces of a cube with the edge equal to 1 in. He cuts the cube into 27 smaller identical cubes. How much more paint does he need in order to paint completely faces of all smaller cubes?

p4. Arrange the whole numbers 1 through 15 in a row so that the sum of any two adjacent numbers is a perfect square. In how many ways this can be done?

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Suppose that we know that for all integer numbers x the value of $ax^2 + bx + c$ is also integer. Can we conclude that the numbers a, b, c are integers?

p2. Let x, y be integer numbers such that $3x + 7y$ is divisible by 19. Prove that $43x + 75y$ is also divisible by 19.

p3. What is the biggest power of 2 which divides the product of 2007 consequitive numbers $2008 \cdot 2009 \cdot 2010... \cdot 4014$?

p4. It is easy to show that the sum of five acute angles of a regular star is equal to 180 degrees. Prove that the sum of five angles of an irregular star is also 180 degrees.

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png

PS. You should use hide for answers.

– Grades 9-12

– **p1.** One throws randomly 120 squares of the size 1×1 in a 20×25 rectangle. Prove that one can still place in the rectangle a circle of the diameter equal to 1 in such a way that it does not have common points with any of the squares.

p2. How many digits has the number 2^{70} (product of 70 factors of 2)?

p3. Prove that the equation $x^2 - 2y^2 = 1$ has infinitely many integer solutions (i.e. there are infinitely many integers x and y which satisfy this equation).

p4. Find all x, y, z which satisfy the following system of equations:

$$\frac{xy}{x+y} = \frac{8}{3}$$

$$\frac{zy}{z+y} = \frac{12}{5}$$

$$\frac{zx}{z+x} = \frac{24}{7}$$

PS. You should use hide for answers.