**Manhattan Mathematical Olympiad 2008**

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– Grades 5-6

– **p1.** What is the minimal number of acute triangles one can cut the regular polygon with $2008$ edges? Justify your answer.

**p2.** A boy has as many sisters as brothers, but each sister has only half as many sisters as brothers. How many brothers and sisters are in the family?

**p3.** The teacher asked each of four children to think of a four-digit number. "Now please transfer the first digit to the end and add the new number to the old one. Tell me your results".
Mary: $8,612$
Jack: $4,322$
Kate: $9,867$
John: $13,859$.
"Everyone except Kate is wrong", said the teacher. How did he know?

**p4.** There are three closed boxes on a table. It is known that one contains two black balls, another contains one black and one white ball, and the third one contains two white balls. Each box has a sticker: "Two whites", "Two blacks", "One white and one black". It is known that all stickers are wrong. How can one place stickers on the boxes correctly by taking just one ball from one box, and not looking inside?

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Heights of the acute triangle $ABC$ intersect at the point $O$. It is known that $OC = AB$. Find the angle $ACB$.

**p2**. Prove that if p and q are prime numbers which are greater than $3$ then $p^2 - q^2$ is divisible by $24$.

**p3.** What is the angle between the hands of a clock when it shows $9:20$ ?

 

**p4.** Prove that there exists a whole number divisible by $5^{1000}$ which does not contain zeros among its digits.

PS. You should use hide for answers.

– Grades 9-12

– **p1.** All trees in the forest have different heights and are taller than $10$ ft and shorter than $50$ ft. It is known that the distance between any two trees does not exceed the difference of their heights. Prove that the forest can be surrounded by a fence $80$ ft long.

**p2.** Recall that the symbol $n!$ means the product $1 \cdot 2 \cdot 3 \cdot ... \cdot n$. Simplify

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{2007}{2008!}.$$

**p3.** Find last two digits of the number $14^{14^{14}}$ (we raise $14$ to the power $14^{14}$).

**p4.** Solve the equation $x^2/3 + 48/x^2 = 10(x/3 - 4/x)$.

PS. You should use hide for answers.