



Manhattan Mathematical Olympiad 2009

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– Grades 5-6

– **p1.** Insert the pluses between (some of) the digits of 987654321 to get the total of 99.

p2. 30 students from five grades donated 40 toys. It is known that students from the same grade donated equal number of toys, while students from different grades donated different number of toys. How many students donated exactly one toy?

p3. A nonstop train leaves New-York for Boston at 60 miles per hour. Another nonstop train leaves Boston for New-York at 40 miles per hour. How far apart are the trains 1 hour before they pass each other? You may assume that the railroad is a straight segment.

p4. A square carpet of the size 4×4 meters contains 15 holes (you may assume that the holes are dots). Prove that one can cut out from it a carpet of the size 1×1 meter which does not contain holes inside.

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Steve and Pete play the following game on the board which is a rectangle 1×10 divided into 10 consecutive squares. There is a coin on the first square. Each player in his turn can move it forward by one, two or three squares. The winner is the one who finishes first by placing the coin on the last (tenth) square. Steve starts first. How he should play in order to win regardless of the moves of his opponent?

p2. Given two rectangular pieces of plywood of size 73×19 centimeters and a pencil how to find the center of one of them?

p3. It is easy to cover 64 squares of the chessboard by 32 dominos, so that each domino covers two squares (the sizes of squares of the chessboard and those of the dominos must match, of course). Can one cover 62 squares of the chessboard with two diagonally opposite squares cut off, by 31 dominos?

p4. Prove that the number $2009k + 3$ is not a cube of any other integer number for any integer k .

PS. You should use hide for answers.

– Grades 9-12

– **p1.** Prove that any pair of integer numbers which satisfies one of the equations $x + y\sqrt{5} = (9 + 4\sqrt{5})^n$ for $n = 0, \pm 1, \pm 2, \dots$ also satisfies the equation $x^2 - 5y^2 = 1$.

p2. A pyramid is based on a 9-gon. Edges of the pyramid and diagonals of the base (but not the edges of the base!) are colored by one of the two colors: blue or red. Prove that there exist three vertices of the pyramid such that the triangle with whose vertices has all its edges colored by the same color.

p3. 44 trees are planted around a circle. At the beginning, 44 birds sit on the trees (one bird on each tree). From time to time two birds simultaneously fly in the opposite directions to the neighboring tree (one flies clockwise the other one counterclockwise). Can they gather on the same tree?

p4. A museum of modern art has the floor plan which is a polygon (not necessarily convex) with 2009 vertices. The manager wants to make sure that every point of the museum is watched by a guard. The guards are stationed at fixed posts, but they are able to turn around. Prove that it suffices to have 669 guards for that purpose.

PS. You should use hide for answers.
