

**Utah Mathematical Olympiad 2015**

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- 1 Three trolls have divided  $n$  pancakes among themselves such that:
- Each troll has a positive integer number of pancakes.
  - The greatest common divisor of the number of pancakes held by any two trolls is bigger than 1.
  - The three greatest common divisors obtained in this way are all distinct.

What is the smallest possible value of  $n$ ?

- 2 In  $\triangle ABC$ ,  $\overline{AC} = 13$  and  $\overline{AB} = 20$ , and the length of the altitude from  $A$  to  $\overline{BC}$  is 12. If  $M$  is the midpoint of  $\overline{BC}$ , find all possible length(s) of  $\overline{AM}$ , and demonstrate that these length(s) are achievable.

- 3 Find, with proof, all positive integers  $n$  with  $2 \leq n \leq 20$  such that the greatest common divisor of the coefficients of  $(x + y)^n - x^n - y^n$  is equal to exactly 3.

- 4 Anastasia and Balthazar need to go to the grocery store, which is 100 km away. Anastasia walks at 5 km/hr, and Balthazar walks at 4 km/hr. However, they also own a single bike, and each of them bikes at 10 km/hr. They are allowed to go forwards or backwards, and the bike will not get stolen if they drop it off along the way for the other person to pick up. What is the shortest amount of time necessary for both of them to get to the grocery store?

- 5 A  $3 \times 3$  grid is filled with integers (positive or negative) such that the product of the integers in any row or column is equal to 20. For example, one possible grid is:

$$\begin{bmatrix} 1 & -5 & -4 \\ 10 & -2 & -1 \\ 2 & 2 & 5 \end{bmatrix}$$

In how many ways can this be done?

- 6 A triangular pyramid with apex  $O$  and base  $\triangle ABC$  has the property that the perimeter of  $\triangle ABC$  is 84. Additionally, one can place a cylinder of radius 4 and height 10 completely inside the pyramid such that one of its bases is in the same plane as  $\triangle ABC$ . What is the minimum possible height from  $\triangle ABC$  to apex  $O$ ? Show that this height is achievable.