Manhattan Mathematical Olympiad 2010

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– Grades 5-6

– **p1.** Number 23 is written on the blackboard. Every minute the following procedure is performed: the written number is erased and the product of its digits plus 12 is written on its place (for instance, after first minute the number $2 \times 3 + 12 = 18$ will be written). What number will be on the blackboard after one hour?

p2. A student took a test consisting of 20 problems. Each correct solution gives him 8 points, for each incorrect solution he gets negative 5 points. For a problem which he did not try to solve he receives 0 points. The student got the total of 13 points. How many problems did he try to solve?

p3. Is it possible to place 25 pennies on a table such that each of them touches exactly three others?

p4. Prove that among all people on earth there are two which have the same number of friends. Note:A is a friend of B if B is a friend of A

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Prove that all numbers 1156, 111556, 11115556, ...are perfect squares.

p2. Let M be the set of integer numbers which can be written as $x^2 + 5y^2$ where x, y are integers. For instance, the number 1 is in M but 2 is not. Prove that the product of two numbers belonging to M also belongs to M.

p3. Is it possible that lengths of all sides of one triangle are less than 1 centimeter, lengths of all sides of another triangle are bigger than 100 centimeters but the area of first triangle is bigger than the area of the second one?

p4. The squares of the 8×8 chess board are colored into 4 colors such that every 2×2 square contains each color exactly once. Prove that the four corners of the chess board are colored in different colors.

PS. You should use hide for answers.

– Grades 9-12

– **p1.** An infinite sequence a_n, $n = 1, 2, 3...$ of numbers is defined by the following conditions: $a_1 = 2$, $a_2 = 3$, $a_{k+2} = a_{k+1}/a_k$for $k \geq 3$. Find a_{2010}.

p2. For any positive integer n show that $1^3 + 2^3 + 3^3 + ... + n^3$ is a perfect square.

p3. You are given a straightedge with notches every 1 cm. Using only it and a pencil construct a straight line perpendicular to a given one.

p4. On an infinite square grid paper finitely many squares are black and the rest are white. We say a square is affected if its color differs from the the color of its two adjacent squares, one below and one to its left (both have to be of same color for that). Every minute all affected squares change their color at once. Prove that in finite time all squares become white regardless of the initial position of black squares.

PS. You should use hide for answers.