Manhattan Mathematical Olympiad 2011

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by parmenides51

– Grades 5-6

– **p1.** Can you cut the figure on the right into three congruent pieces and then put them back together to form one regular hexagon?
https://cdn.artofproblemsolving.com/attachments/3/d/aee095b1cb03b98248fe1afcfcfe637d3ca03
png

p2. John and Mary play in a very long chess tournament. John plays every 16th day, while Mary plays every 25th day. Will they sometime have to play in two consecutive days?

p3. Prove that at least one of any 18 successive three-digit numbers is divisible by the sum of its digits.

p4. 30 children from school went to a museum in pairs. After visiting the museum they went back to school also in pairs (possibly different). Show that upon their arrival back to class it is always possible to divide them into three groups such that in any group no two kids were a pair on either way.

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Find the smallest positive integer number which has the following two properties:
a) one digit is 0;
b) after deleting this 0 the number becomes 9 times less.

p2. A Perfect Tomatoes manufacturer packs tomatoes into two types of cans: long narrow cylinder and a wide flat cylinder (see picture). Both contain 7 perfectly round shaped tomatoes of equal size touching each other and the walls. The remaining space between the tomatoes is filled with brine. Which of the two cans requires more brine?
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png

p3. Prove that if the the numbers n and $n^2 + 2$ are prime then the number $n^3 + 2$ is also prime.

p4. A square is cut into a number of rectangles in such a way that no point of the square is a common vertex of four rectangles. Prove that the number of points of the square that are the vertices of rectangles is even.

PS. You should use hide for answers.

– Grades 9-12

– **p1.** You can do either of two operations to a number written on a blackboard: you can double it, or you can erase the last digit. Can you get the number 14 starting from the number 458 by using these two operations?

p2. Show that the first 2011 digits after the point in the infinite decimal fraction representing $(7+\sqrt{50})^{2011}$ are all 9's.

p3. Inside of a trapezoid find a point such that segments which join this point with the midpoints of the edges divide the trapezoid into four pieces of equal area.

p4. New chess figure called "lion" is placed on the 8×8 chess board. It can move one square horizontally right, or one square vertically down or one square diagonally left up. Can lion visit every square of the chess board exactly once and return in his last move to the initial position? You can choose the initial position arbitrarily.

PS. You should use hide for answers.